Building a Better Roller Coaster

CC Edwards

CALCULATORS: Casio: *ClassPad 300 • ClassPad 300 Plus • ClassPad Manager*
TI: *TI-89, TI-89 Titanium • Voyage 200*

The Casio ClassPad 300

You are asked to design the first ascent and drop for a new roller coaster. By studying photographs of your favorite coasters, you decide to make the slope of the ascent 0.8 and the slope of the drop -1.6.

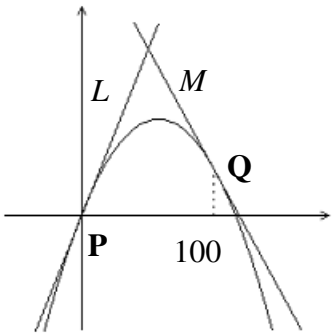
You also decide to connect these two straight stretches, $y = L(x)$ and $y = M(x)$, with part of a parabola $f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in feet.

Assuming that the horizontal distance between P and Q is 100 feet:

(a) Find a formula for $f(x)$.

(b) Plot L, f, and M to verify that the transition is smooth.

(c) Find the difference in elevation between P and Q.



Interpreting and Setting Up the Problem

For the track to be smooth there can't be abrupt changes in direction, so you want the linear segments L and M to be tangent to the parabola at the transition points **P** and **Q**. And since the horizontal distance between **P** and **Q** is 100 feet, you should place point **P** at the origin, as pictured at the right.

What's needed to solve the Problem?

Placing **P** at the origin simplifies the problem in three ways:

- The equation of the ascent line L is $L(x) = .8x$ since this line passes through the origin and has slope 0.8.
- The parabola takes the form has the form $f(x) = ax^2 + bx$ since it passes through the origin.
- The equation of the decent line M, which has slope -1.6 and passes through point **Q**, has the form $M(x) = -1.6x + d$ where $M(100) = -160 + d$.

So the problem boils down to finding a, b, and d. What information helps us solve for these three variables? Well here's what we have:

- $f'(0) = 0.8$ because the line L is tangent to the parabola at $x = 0$.
- $f'(100) = -1.6$ because the line M is tangent to the parabola at $x = 100$.
- **Q** is also on the parabola, so $M(100) = f(100)$. Hence d = $f(100) + 160$.

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The Casio ClassPad 300

Here's how we get the ClassPad to solve the problem:

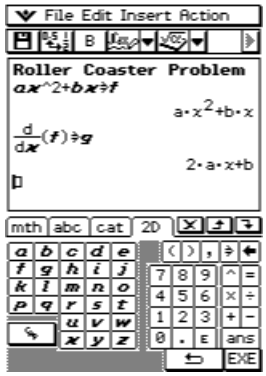
Start an eActivity.

1. On the Silk Screen Menu Bar tap **MENU**.
2. Tap the **eActivity** icon.
3. On the Menu Bar tap **File/New**. If you're asked to clear the screen, tap **OK**.
4. Enter a title:
   - Press **Keyboard** and tap **abc**.
   - On the Toolbar, tap **B** (for bold).
   - Enter **Roller Coaster Problem** by tapping the appropriate letters on the keypad.
   - Tap **EXE** when finished.

Find a, b, and d:

1. Define the function $f(x) = ax^2 + bx$:
   - On the Toolbar, tap the first drop down menu arrow ▼ and tap the **evaluation symbol** (Integral with pen).
   - On the Keyboard, tap **mth**.
   - Enter $ax^2 + bx$ and assign it to the variable *f*.
     VAR a *x* ^ 2 + b *x* ⇒ *f* EXE

2. Assign the derivative of ***f*** to the variable *g*:
   - Tap **2D** . If ⇦ is displayed on the bottom line, tap it to display the 2D menu. If ▼ instead of ▲ is displayed in the middle of the bottom line, tap ▼ to display the second part of the 2D menu.
   - Tap the **derivative symbol** (middle of the bottom row).
   - Tap ***x*** .
   - Tap the box in parentheses and then tap **VAR f** .
   - Tap on the screen to the right of the parentheses surrounding *f* .
   - Tap ⇒ **g EXE** .

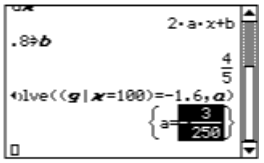


3. Since $b = f'(0) = 0.8$, assign the value 0.8 to the variable *b*.
   - **8 ⇒ b EXE**

The Casio ClassPad 300

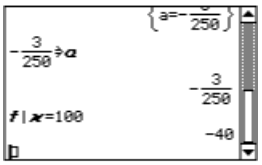
4. Use the fact that $f'(100) = -1.6$ to solve for a :

- On the Menu Bar tap **Action/Equation/Inequality/solve**.
- In parentheses enter g evaluated at 100.
  (g mth OPTN ▌ x = 1 0 0)
- Set this equal to -1.6. **= (-) 1 • 6**
- Tell the ClassPad to solve for the variable a. **, VAR a) EXE**

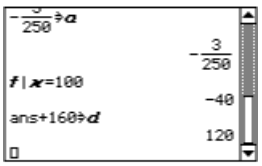


5. Assign the value found in Step 4 to the variable a:

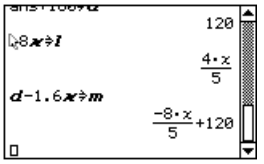
- Highlight the value found for a and drag it to the box in the lower left corner.
- Place the cursor after this number and tap **⇒ a EXE**.



6. Find the vertical distance, $f(100)$, between **P** and **Q**.
f OPTN ▌ x = 1 0 0 EXE
Answer to part c: Q is obviously below **P**, so the vertical distance is 40.



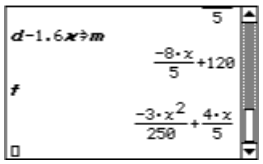
7. Evaluate $d = f(100) + 160$ and assign the result to the variable d.
ANS + 1 6 0⇒ VAR d EXE



Find and Graph Functions *L*, *f*, and *M*:

1. Define the linear functions L and M:

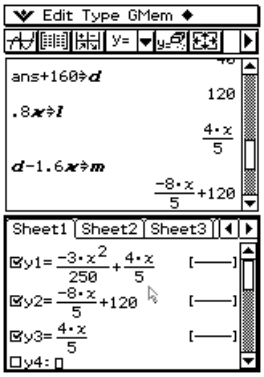
- Define L by assigning $0.8x$ to variable l. **• 8 x ⇒ ▌ EXE**
- Define M by assigning $-1.6x + d$ to variable m.
  d – 1 . 6 x ⇒ m EXE



2. Display f. **f EXE**
Answer to part a: $f(x)$ = the equation displayed on the screen.

3. Place functions L, M, and f in the Graph Editor:

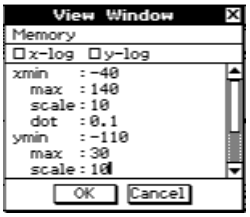
- On the Menu Bar tap **Insert/Graph Editor**.
- Drag the definition of f to y1 in the Graph Editor; Drag the definition of m to y2: Scroll up and drag the definition of l to y3.
- Select functions y1, y2, and y3 by tapping the box to the left of the function to place a check mark in the box.

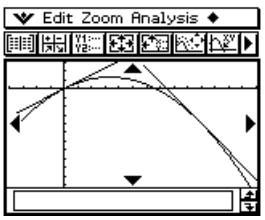


4. Graph these functions:

- Tap on the Toolbar.
- On the Menu Bar, tap **/Settings/View Window**.
- Set the window as in the first picture at the right.
- Tap **OK** to display the graphs.

Answer to part b: As the picture at the right shows, transition between the curves is smooth.





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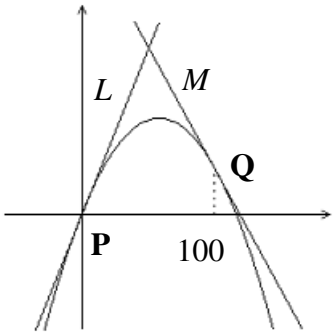

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The TI-89

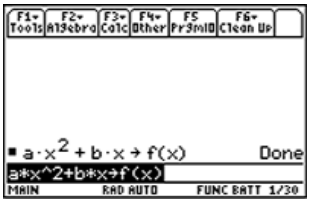
Here's how we get the TI-89 to solve the problem:

Find *a*, *b*, and *d*:

1. Define the function $f(x) = ax^2 + bx$:

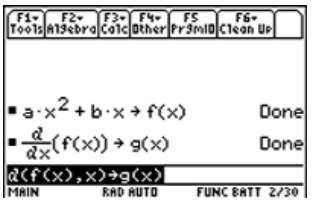
On the Home screen, store $ax^2 + bx$ in $f(x)$ by pressing:

- **alpha A** to insert the letter a
- × **X** ∧ **2** to multiply a by x^2
- **+ alpha B** to enter + *b*
- × **X** to multiple *b* by *x*, and
- STO▶ **alpha F (X) ENTER** to store ax^2 in $f(x)$.



2. Assign the derivative of *f* to the variable *g*:

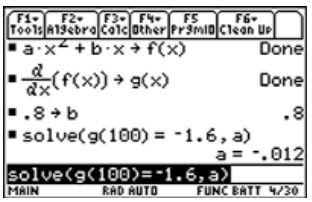
- Press **F3 1** to start a derivative.
- Enter $f(x)$ by pressing **alpha I (X)** .
- Press **,** .
- Press **X** to tell the calculator that the derivative is taken with respect to *x*.
- Press **)** to tell the calculator you are finished entering the derivative.
- Press STO▶ **alpha G (X) ENTER** to store the derivative in $g(x)$.



3. Since $b = f'(0) = 0.8$, assign the value 0.8 to the variable *b*. Press **. 8 STO▶ alpha B ENTER**)

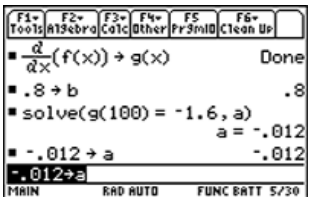
4. Use the fact that $f'(100)$ to solve for *a*:

- Press **F2 1** to enter the solver.
- Enter $g(100) = -1.6$. (**alpha 7 (1 0 0) = (-) 1 . 6**)
- Enter **, alpha =) ENTER** to solve for *a*.



5. Assign the value found in Step 4 to the variable *a*:

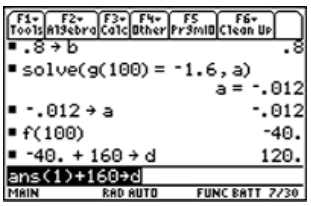
- Press ∧ **ENTER** to place the last answer on the command line.
- Press **2nd ◀ ▶ ▶ ← ←** to erase ***a*** =.
- Press **2nd ▶ STO▶ alpha A ENTER** to store the value in a.



The TI-89

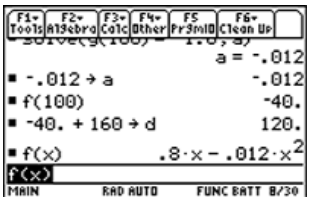
6. Find the vertical distance, ***f***(100), between **P** and **Q**.
alpha F (1 0 0) ENTER

Answer to part c: **Q** is obviously below **P**, so the vertical distance is 40.



7. Evaluate ***d*** = *f*(100) + 160 and assign the result to the variable *d*.
2nd ANS + 1 6 0 STO▶ alpha D ENTER

Find and Graph Functions *L*, *f*, and *M*:

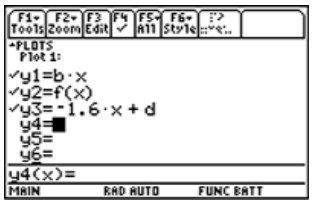


1. Display *f*. **alpha F (X) ENTER**

Answer to part a: *f*(*x*) = the equation displayed on the screen.

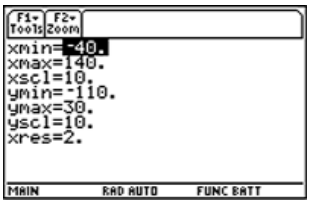
2. Place functions *L*, *M*, and *f* in the Graph Editor:

- Press ◆ **Y=** to enter the Y= editor.
- Uncheck or delete all functions and stat plots.
- Place the cursor after y1 = and press **ENTER** .
- Enter *bx* to define line *L*. **alpha B × X ENTER**
- Enter the definition of *f*(*x*). **ENTER alpha F (X) ENTER**
- Define line *M* by entering -1.6*x* + *d* in y3.
  ENTER (-) 1 . 6 × X + alpha D ENTER



3. Graph these functions:

- Press ◆ **F2** to enter the Window editors and set the window as it appears in the first picture at the right.
- Press ◆ **F3** to graph the functions.



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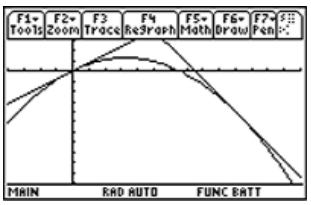

THE CASIO ADVANTAGE

- Ease of Use: It is easier to enter equations and expressions on the *ClassPad 300.*
- On the *ClassPad*, you can graph an already defined function by dragging it to the Graph editor. On the TI-89, you must manually enter in the Y= editor.
- On the *ClassPad*, you can change your view of the graph simply by tapping one of four arrow keys. On the TI-89, you must leave the screen and adjust in the Window editor.