Newton's Law of Cooling
Using the ClassPad 300

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On a cold 26°F December day in Chicago, Detective Daniels went to a large apartment complex to investigate a murder. When he arrived at noon, Sergeant Spencer said that there were many suspects, but they were having trouble narrowing the list since they didn't know the exact time of death. Detective Daniels took out a thermometer and measured the temperature of the body, finding it to be 77.9 °F. He also noted that the thermostat in the room was set at 72°F. An hour later at 1:00 P.M., he found the body temperature to be 75.6°F. He then left for lunch announcing that when he returned, he would tell them when the murder was committed. How did he plan do this?

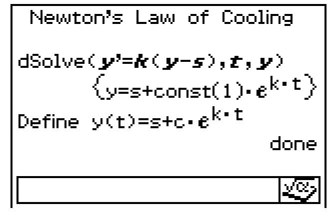
At first it looks like Detective Daniels doesn't have enough information to find the time the murder was committed. But Detective Daniels remembered seeing Newton's Law of Cooling when he took calculus. Unfortunately he didn't remember precisely what the law was. So when he went home for lunch he borrowed his daughter's calculus book and looked it up. Here's what he found:

> **Newton's Law of Cooling:** The rate at which the temperature of an object changes is proportional to the difference between the temperature of the object and the temperature of the surroundings.
>
> **Mathematically,** this says $\frac{dy}{dt} = k(y - s)$ where:

$t =$ time
$y(t) =$ temperature of the object at time t
$s(t) =$ temperature of the surrounding area at time t
$k =$ the constant of proportionality

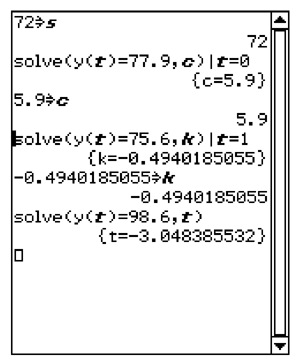
After reading this, Detective Daniels realized that he had forgotten how to solve a differential equation. So he asked his daughter for help. His daughter knew that there was no way she could give her dad a refresher course in differential equations in 15 minutes, so she showed him how to solve the problem using her ClassPad.

First she tapped A on the Icon Panel and started a new eActivity by tapping **File/New/OK**. Then she wrote a program for solving Newton's Law of Cooling. Here's how you can write the same program:



- Create a title. (optional)
- Tap the drop down arrow next to u on the Toolbar and tap the icon in order to enter mathematical expressions.
- Solve the differential equation for Newton's Law of Cooling. (**dSolve** and the prime for the first derivative are housed in the 9 **CALC** menu on the Soft keyboard.)
- Define the solution as the function y(t). (The **Define** command is housed in the (menu on the Soft keyboard. To find it tap the drop down arrow under **Form** and then tap **Cmd**. Tap the letter **D** at the bottom of the screen and then tap the **Define** command. Finally, tap **INPUT** to place this command on the screen.)
- Insert a Main window in which to solve the problem. (Tap **Insert** on the Menu bar and then tap **Main** in the drop down menu.)
- Save the eActivity for future use (optional).

Then she told her Dad to do the following:

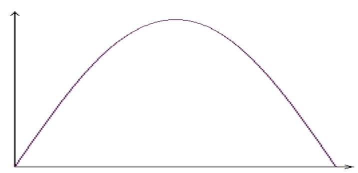


- Store the surrounding temperature 72^o in the variable ***s***.
- Time $t = 0$ occurred at noon when the body temperature was found to be $77.9\ ^o$F. This will allow us to solve for ***c*** as shown in the second line of the figure at the right.
- Store the solution for ***c*** in the variable ***c.***
- Solve for ***k*** by using the fact that at 1 p.m. the temperature of the body was 75.6^oF. (This appears on the fourth line of the figure at the right. The "with" command, | , is found in the 9 **OPTN** menu on the Soft keyboard. The complicated solution for ***k*** was simplified by highlighting it an then tapping the icon on the Toolbar.)
- Store the solution for ***k*** in the variable ***k.***
- Find when the murder was committed by solving for ***t*** when the body was at normal body temperature, which is 98.6^oF.

CONCLUSION: $t \approx -3.05$ indicates that the murder occurred at 9 a.m., approximately 3 hours before noon.

EXERCISE: At noon next day, Detective Daniels arrived on the scene of another murder. But this one took place in a vacant lot. He took the temperature of the body and found that it was 60^{o}F. He also noted that the outdoor temperature was 26^{o}F. An hour later he found that the temperature of the body had dropped to 40^{o}F. He then called the weather bureau to find out what the temperature was at 9 a.m., being informed that it was 18^{o}F. Using his daughter's ClassPad, he then found the time of the murder. What time did he find?

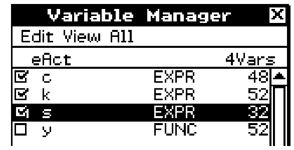
COMMENT: Note that in the great outdoors, the surrounding temperature is *not* constant; it is a function of time. One half the period of *some* sine function is usually used to model temperature during a 24 hour period of time. But $x = 0$ is not midnight since the coldest temperature of the night usually occurs between 2 and 5 a.m. However, when looking at the change in temperature over a period of only a few hours, as you are in this exercise, it is safe to assume a linear model for time.



SOLUTION:

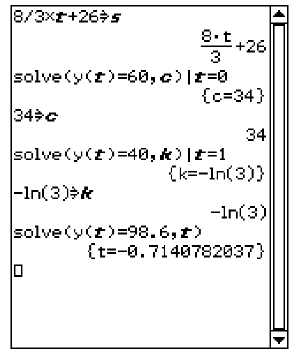
Start a new Main window in the eActivity by tapping **Insert** on the Menu bar and then tap **Main** in the drop down menu.

Clear out the values you may have saved the last time you used this eActivity by first tapping **0/Settings/Variable Manager/eAct/eAct/**, then tapping the variables ***c***, ***k***, and ***s***, and finally tapping **Edit/Delete/OK/Close/Close.**



Since $t = 0$ occurs at noon, the linear model for the outside temperature ***s*** passes through the points $(-3, 18)$ and $(0, 26)$. So the equation for ***s*** is $s = \frac{8}{3}t + 26$. Store this value in ***s*** and then solve Newton's Law of Cooling.

From here, all you have to do is follow steps in the previous example to solve the problem. The figure the right shows you what this looks like.



Since $t \approx -0.7$ hours, the murder took place approximately 42 minutes before noon. That is, at 11:18 a.m.