Solving Linear Equations: Part 3

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Casio ClassPad 300 & ClassPad Manager Software

Suppose you wanted to solve the two equations $2x + 3y = 5$ and $3x - 5y = 7$ simultaneously, i.e. you wanted to find all pairs (x,y) of values which satisfied both equations. The problem can be formulated in vector-matrix form as

$$\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

The solution to our problem then is given by

$$\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

where the 2 by 2 matrix on the left hand side of the equation is above is the inverse of the 2 by 2 matrix on the left hand side of the first equation. Thus, these two matrices have the property

$$\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where the matrix on the right hand side is the so-called identity matrix. Not every 2 by 2 matrix has an inverse matrix but the one we are working with does, and the ClassPad will give it to us. Note that if we have the inverse, the following is true.

$$\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix} \text{ so that}$$

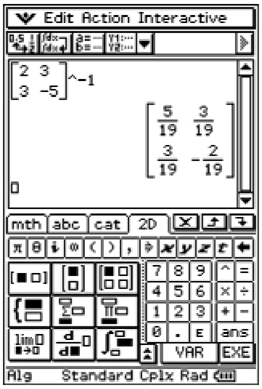
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix} \text{ and finally since } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix},$$

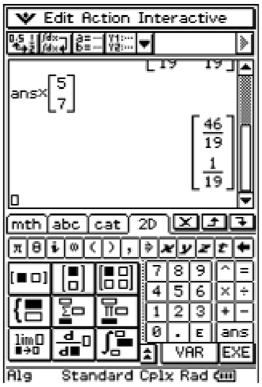
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Thus, if we have the inverse matrix, solving our system of equations involves only a simple matrix multiplication. The following screen captures show one way to carry out these calculations on a ClassPad.

Note that the simplest way to enter a vector or a matrix on the ClassPad is to use the 2D menu. Note also that the solution is presented as a so-called column vector, in which the top entry is the value of x and the bottom entry is the value of y.

You can see that the ClassPad gives an exact, rational solution to the system of equations.





Now what happens if the system has no solution? Something must go wrong, and the only candidate is that the 2 by 2 matrix must not have an inverse.

Here is an example. The system $x + 2y = 4$ and $2x + 4y = 10$ has no solution. The screen capture below shows how the ClassPad tells us that the matrix fails to have an inverse. In fact, there are actually two situations in which the 2 by 2 matrix will fail to have an inverse.

The first, as we have seen occurs when the system of equations has no solution; the second occurs when the system of equations has infinitely many solutions, because, in effect, the two equations define the same line in the plane. An example of this behavior is given by: $x + 2y = 4$ and $2x + 4y = 8$. These equations define the same 2 by 2 matrix as before, but this time if $(x, y) = (x, 2 - x/2)$, both equations are solved.