

Exercise 1: Linear Equations

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Casio ClassPad 300 & ClassPad Manager Software

For b a real number, consider the system of linear equations:

$$x + y + z = 5, x + 4y - 8z = 1, 2x + 3y - z = b$$

- a) Find all values of b such that the system has a solution by carrying out step-by-step Gaussian elimination on your ClassPad to reduce an appropriate augmented matrix. (Do NOT use rref. It will not give you what you need.) You will be able to read off the answer to the question if you remember the significance of a row in an augmented matrix all of whose entries to the left of the dotted line are zero.
- b) For each value of b found in (i), find all solutions to the given system of equations. [HINT: There are at least two ways to do this problem. For each b you find in part a), you can complete the Gaussian Elimination process. Or, you can go back to the beginning of the Gaussian Elimination process, plug in the value of b and use the rref function.]
- c) For each value of b found in (i), find all solutions to the given system of equations with $x = y$. [NOTE: There may not be any such solutions.]

HINT for part c): There are at least two ways to do this problem. First, for each b you find in part a), you can use your answer to part b), which should be of the form:

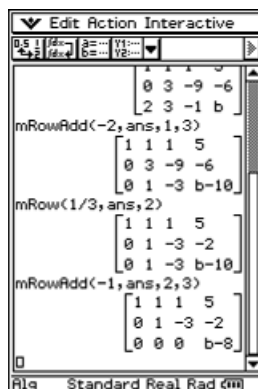
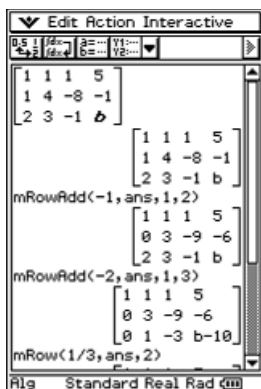
$$x = x_0 + x_1t, y = y_0 + y_1t, z = z_0 + z_1t$$

Then set $x = y$ and solve for t . The rest is easy. Alternatively, you can simply add the linear equation $x - y = 0$ to the system of equations and solve this new system of four equations in four three unknowns.

- d) For each value of b found in (a), find all solutions to the given system of equations with $3x = -4y$. [NOTE: There may not be any such solutions.]

SOLUTIONS:

- a) The following overlapping screen captures show that unless $b = 8$, the system of equations is inconsistent. Note the third row in the last step shown in the Gaussian Elimination process.

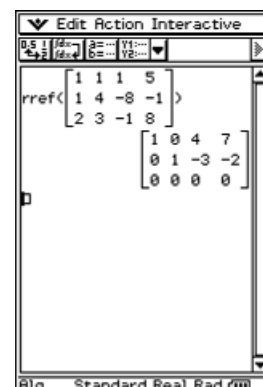


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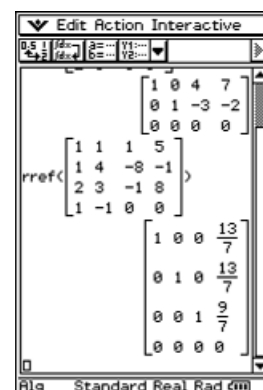
(continued)

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- b) The screen capture at the right gives the reduced form of the augmented matrix associated with the given system of equations (when $b = 8$). It follows that the solutions to the system are given by: $x = 7 - 4t$, $y = -2 + 3t$, $z = t$ for any value of t .



- c) If we add the equation $x = y$ to the given system of equations (when $b = 8$) and apply Gaussian Elimination, we obtain the reduced form shown at the right which shows that the solution is $x = y = 13/7$, $z = 9/7$.



- d) If we add the equation $3x = -4y$ to the given system of equations (when $b = 8$) and apply Gaussian Elimination, we obtain the reduced form at the right which shows that there is no solution to the given system of equations which also satisfies $3x = -4y$.

