Activity 5: Exploring Complex Numbers

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CALCULATOR: Casio: *fx-115ES*

Teaching Notes and Solutions

Objectives: Students will demonstrate the ability to perform mathematical calculations involving complex numbers using the Casio fx-115ES.

Getting Started:

Real numbers are defined as the set of rational and irrational numbers. As we continue our exploration of mathematics, it is important to realize that there is a set of numbers, which extends beyond the real number system. These numbers are referred to as complex numbers. A complex number is defined as a number in the form of $a + bi$ where a and b are both defined as real numbers.

However, i denotes an imaginary number. So, in the form of $a + bi$, a is the real part of the complex number while bi is the imaginary part. In some ways, one might look at a complex number and make a connection with an algebraic binomial. In this sense, complex numbers can operate just like a binomial providing those operations follow the sum, product, and equality of complex numbers

Answers:

1. $12 - 5i$

2. $13 + 14i$

3. 32 + 24i

4. $\frac{18}{97} + \frac{8}{97}i$

5. When $(3 + 2i)$ is squared, it equals $(5 + 12i)$. When $(3 - 2i)$ is squared, it equals $(5 - 12i)$. When we multiply $(5 + 12i)$ by $(5 - 12i)$ it equals 169.

6. Given the problem, $(3 + 5i)(8 + 7i)$, Vicki obtained an answer of $(24 + 12i)$ which is incorrect. It appears as if Vicki multiplied the real part of this complex number and added the imaginary part. However, to multiply these complex numbers correctly, she must FOIL them. Therefore, Vicki must multiply $(3 \bullet 8)$, then $(3 \bullet 7i)$, then $(5i \bullet 8)$, and then $(5i \bullet 7i)$. Those products equal $24 + 21i + 40i + 32i^2$. Because $i^2 = -1$, $32i^2 = -32$. Then, we can simply the product to be $24 + 21i + 40i - 32$ to equal $-11 + 61i$.

7. The conjugate of $(2 + 3i)$ is $(2 - 3i)$ and the product equals 13.

Name _____ **Date** _____

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Student Worksheet Activity 5

There is always a balance between understanding how any mathematical process or function works and incorporating technology into problem solving. It is important for students to understand the properties of complex numbers and the relationship between i, i^2, i^3, i^4, and so on. However, once a student has gained the necessary understanding of these relationships as well as how to operate with complex numbers, the student can use technology to verify their computation and/or enhance their problem solving based upon the integration of such technology.

Calculator Notes:

To Solve Problems Involving Complex Numbers:

- Turn the calculator **ON** .
- Press **MODE** .
- Press **2** for Complex Numbers (CMPLX)
- To enter a complex number using i, press **ALPHA** followed by the **ENG** key. The i is written in red above the ENG key on the right side.

Note: When entered the i, you must press the key twice in order to enter it onto the display.

Problems:

Simplify the following problems using the calculator.

1. $(5 + 4i) + (7 - 9i)$

2. $(2 + i)(8 + 3i)$

3. $(6 + 2i)2$

4. $\dfrac{2}{(9 - 4\text{i})}$

5. Explain why $(3 + 2i)2(3 - 2i)2 = 169$.

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6. Given the problem, $(3 + 5i)(8 + 7i)$, Vicki obtained an answer of $(24 + 12\text{i})$. Is this correct? If it is not correct, explain what steps Vicki did incorrectly and what is the correct answer?

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7. What is the conjugate of $(2 + 3i)$ and what is the product when you multiply $(2 + 3i)$ by its conjugate?

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