

Activity 6: Exploring Exponents

John Neral

CALCULATOR: Casio: *fx-115ES*

Teaching Notes and Solutions

Objectives: *Students will demonstrate the ability to explore patterns involving exponents, solve problems involving exponents, and explore the properties of exponential functions.*

Getting Started:

Have students examine a table of values for an exponential function. Using the table function of the *fx-115ES*, have them input the function $f(x) = 2^x$. Set a start value of -5 and an ending value of 5 with a step increment of 1 . Ask: *What does the shape of an exponential function look like? When you plot the points on a coordinate plane it doesn't possess the same properties as a linear function or absolute value function.*

Students might say that it has some similar qualities of a quadratic function but it is not symmetrical. Nevertheless, an exponential function is rooted in a solid understanding of how exponents work. Explain that an exponent, by its definition, represents the number of times the base is used as a factor. For example, if we were to write 3^4 as a product of factors, we would write $4 \times 4 \times 4 \times 4$ and in simplified form, $3^4 = 81$.

Ask: *What happens when the exponent is a negative integer? When you evaluate a number in standard form, what happens to the units digit as a positive exponent increases?* Throughout this activity, students will explore various properties of exponents and examine a table of values to view its impact on an exponential function.

Answers:

The first eight powers of 3 are $3, 9, 27, 81, 243, 729, 2187, \text{ and } 6561$. The pattern in the units digit is $3, 9, 7, 1$. To calculate 3^{173} , divide the exponent, 173 , by 4 to obtain an answer of $43 \text{ R}1$. The pattern repeats itself 43 times and the remainder 1 indicates that the units digit of 3^{173} is in the first position of the pattern – the number 3 .

- $2^{-3} = 1/8, 2^{-2} = 1/4, 2^{-1} = 1/2, 2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8$.
- Yes, $2^4 = 4^2 = 16^1$. $2^4 = 16$ as does 4^2 .
- $10^1 = 10, 10^2 = 100, 10^3 = 1,000, 10^4 = 1,000, 10^5 = 100,000$
- $(11 + 8)^2 \neq 11^2 + 8^2$. The answer is false.
- To calculate the amount of money received on the 30th day, you could evaluate the function $2^{(X-1)}$ where $x = 30$. When you evaluate $2^{(30-1)}$ it equals 536870912 pennies which is $\$5,368,709.12$. To calculate the total amount of money during the month you would need to calculate:
 30
 $\sum_{x=1} (2^{(x-1)}) = 1073741823$ pennies which equals $\$10,737,418.23$.

Activity 6: Exploring Exponents

CALCULATOR: Casio: *fx-115ES*

Student Worksheet Activity 6

Examine the first eight powers of 3. Using the calculator create a table of values to determine a pattern within the units digit. Then, use the pattern to determine the units digit for 3^{173} .

Calculator Notes:

To Input a Function and Generate a Table of Values:

- Turn the Calculator **On** .
- Press **MODE** followed by **7** for Table.
- Input the function after $f(x)=$.
- Set the Start value and press $=$.
- Set the End value and press $=$.
- Set the Step value and press $=$.
- Examine the table of values.

To Solve a Problem Involving Exponents:

- Turn the calculator **On** .
- Press **MODE** followed by **1** for COMP.
- Press **SHIFT SETUP** followed by **1** for the MthIO (Math Input Output format).
- Enter the base.
- Press the **exponent key** to use the exponent template.
- Enter the exponent.

Sample Problems:

1. Evaluate 2^x where $-3 \leq x \leq 3$.
2. Does $2^4 = 4^2 = 16^1$? Explain.
3. Evaluate 10^x where $1 \leq x \leq 5$. What pattern do you notice between the number in exponential form and the number written in standard form?
4. Let $x = 11$ and $y = 8$. Examine the following question.
True or False: $(x + y)^2 = x^2 + y^2$.
5. If you were given one penny on the first of the month and told that the amount of money would double each day until the end of the month (let's say that there are 30 days in the month) how much money would you receive on the 30th day? What formula would you use to determine the amount of money received on any particular day? How much money would you receive in total for all 30 days of the month?