

Middle Grades Activity: Nested Fractions

CALCULATOR: Casio *fx-300ES*

INTRODUCTION:

A nested fraction is a fraction that contains other fractions within its numerator and/or denominator, often forming some kind of pattern. A nested fraction is sometimes called a telescoping fraction. Examples are:

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}} \quad \text{and} \quad \frac{\frac{2}{3} + 4}{2 + \frac{4}{3}}$$

Notice, as in the case of the first example, that the nesting of some fractions can continue forever.

PROCEDURE:

Turn the calculator **[ON]**.

Make sure the *fx-300ES* is set up correctly for this activity by pressing the following key sequences:

[SHIFT]-[SETUP] [1] to enter Math Input/Output mode;

[SHIFT]-[SETUP] [8] [1] to enter Normal Display 1;

[SHIFT]-[SETUP] [] [2] to force fractions to display as improper fractions.

Type $\left[\frac{X}{X}\right]$ **[1][] [2] [=]**. The *fx-300ES* will display the fraction $\frac{1}{2}$. This is an example of using the fraction key ($\left[\frac{X}{X}\right]$) to input fractions using Casio's "Natural Display."

Press **[]**, which will allow editing of the most recent expression entered, with the cursor placed at the *end* of the expression. (Pressing **[]** instead would place the cursor at the *beginning* of the expression.)

Now press **[]** to enter the denominator of the fraction $\frac{1}{2}$, then type:

$$\left[-\right] \left[\frac{X}{X}\right] \left[1\right] \left[] \left[2\right] \left[=\right]$$

The *fx-300ES* outputs the fraction $\frac{2}{3}$. Even though the display only shows the entered expression as $2 - \frac{1}{2}$, the next step will verify that a nested fraction was indeed calculated:

Press **[]** to edit the last entry. (Notice the nested fraction $\frac{1}{2 - \frac{1}{2}}$). Then type:

$$\text{[] [] [-] [\frac{X}{X}] [1] [] [2] [=]}$$

The *fx-300ES* outputs the fraction $\frac{3}{4}$.

Pressing the **[]** and **[]** keys allow you to “Replay”, or cycle through, previous entered expressions and their calculated values.

Exercise 1. Press **[]** enough times to return to the most recent entered expression, whose shown value is $\frac{3}{4}$. Continue typing the sequence:

[] [] ... (until the cursor is to the right of the “2” in the existing expression)

$$\text{[-] [\frac{X}{X}] [1] [] [2] [=]}$$

Write the next two outputs in the pattern created by the nested fractions.

The *fx-300ES* can only nest a total of five fraction bars due to the size of its display. We cannot explore this pattern any further using the calculator; however, we have already seen enough outputs to be able to form some conclusions:

Exercise 2. The first five outputs generated by the nested fraction

$$2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \dots}}}}$$

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}.$$

What will the next five outputs be?

SOLUTIONS TO EXERCISES:

Exercise 1. The first three outputs were $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$. The next two are:



Exercise 2. The next five outputs are:



To obtain a new output, simply increase the numerator and denominator each by 1.

If we continue this pattern infinitely, we will eventually reach an output of 1!
We can trick the *fx-300ES* into showing this by entering the fraction $\frac{999999999999}{1000000000000}$, then pressing [=]. (That's twelve 9's in the numerator, and twelve 0's in the denominator.)