

Middle Grades Activity: Absolute Value and Distances on the Number Line

CALCULATORS: Casio *fx-300ES*

INTRODUCTION:

This activity allows students to establish the idea that absolute value should be equated with the concept of *distance*. In addition, it teaches the correspondence between a distance and a *difference* (subtraction) of two real numbers.

PROCEDURE:

Turn the calculator **[ON]**.

Make sure the *fx-300ES* is set up correctly for this activity by pressing the following key sequences:

[SHIFT]-[SETUP] [1] to enter Math Input/Output mode;

[SHIFT]-[SETUP] [8] [1] to enter Normal Display 1;

[SHIFT]-[SETUP] [] [2] to force fractions to display as improper fractions.

Consider the following number line:

Type "1 ÷ 3" , then press **[=]**. The *fx-300ES* will display the fraction $\frac{1}{3}$.

(This is because the calculator is set up in the *d/c* fraction display mode.)

To force the *fx-300ES* to display a decimal, press the **[S D]** key.

Notice that the fraction $\frac{1}{3}$ is equivalent to the repeating decimal 0.333333...

Now let's investigate the other repeating decimals comprised of only a single-digit repetend:

Exercise 1. What fraction will the repeating decimal $0.\overline{6} = 0.666666\dots$ be equivalent to?

Type "0.4444..." until the top line of the screen is filled with 4's. Then press **[=]**. Notice that the *fx-300ES* interprets this as a repeating decimal, and outputs $\frac{4}{9}$.

Exercise 2. Find the decimal that, when typed into the calculator, outputs the fraction $\frac{7}{9}$.

Exercise 3. Use your fx-300ES, and any patterns you may observe, to fill in the following blanks. Your answers should be fractions in simplest form.

$$0.\bar{1} = \underline{\quad\quad} \quad 0.\bar{2} = \underline{\quad\quad} \quad 0.\bar{5} = \underline{\quad\quad} \quad 0.\bar{8} = \underline{\quad\quad} \quad 0.\bar{9} = \underline{\quad\quad}$$

What about repetends that involve more than one digit?

Type “7 ÷ 11” , then press [=]. The fx-300ES will display the fraction $\frac{7}{11}$.

Use the [S D] key to display the decimal 0.636363..., which can be written in

shorthand as $0.\overline{63}$. (Note that the calculator rounds the last displayed digit of this

decimal to a 4 rather than a 3. Watch out for this! Just because the last digit doesn't

fit the pattern won't necessarily mean that the decimal doesn't repeat.)

Type “0.202020..” until the top line of the screen is filled. Then press [=].

Notice that the fx-300ES interprets this as a repeating decimal, and

outputs $\frac{20}{99}$.

Exercise 4. Find three more fractions that are equivalent to decimals with 2-digit repetends.

Exercise 5. Find a fraction that is equivalent to a decimal with a 3-digit repetend.

SOLUTIONS TO EXERCISES:

Exercise 1. Since $0.\bar{3} = \frac{1}{3}$, and $0.\bar{6} = 2 \times (0.\bar{3})$, $0.\bar{6}$ must equal

Exercise 2. Typing 0.3333333... results in an output of $\frac{1}{3}$ (which is equivalent to $\frac{3}{9}$), and typing 0.4444444... results in an output of $\frac{4}{9}$. Since $\frac{7}{9} = \frac{4}{9} + \frac{3}{9}$, we must type

$$0.3333333... + 0.4444444... = \boxed{0.7777777...}$$

Exercise 3. The correct fractions are as follows:

$$0.\bar{1} = \boxed{\frac{1}{9}} \quad 0.\bar{2} = \boxed{\frac{2}{9}} \quad 0.\bar{5} = \boxed{\frac{5}{9}} \quad 0.\bar{8} = \boxed{\frac{8}{9}} \quad 0.\bar{9} = \frac{9}{9} = \boxed{1}$$

Notice that the last answer indicates that 0.9999999... is actually equal to 1!

Exercise 4. The only fractions that will work are those that, in reduced form, have denominators of 11, 33, or 99. For example:

$$\frac{3}{11} = 0.\overline{27} \quad \frac{19}{33} = 0.\overline{57} \quad \frac{78}{99} = 0.\overline{78}$$

Along with denominators 3 and 9 (which create one-digit repetends), these numbers form the factors of 99.

Exercise 5. The only fractions that will work are those that, in reduced form, have denominators of 37, 111, 333, and 999. For example:

$$\frac{15}{37} = 0.\overline{405} \quad \frac{82}{111} = 0.\overline{738} \quad \frac{182}{333} = 0.\overline{546} \quad \frac{400}{999} = 0.\overline{400}$$