

# Middle Grades Activity: Repeating Decimals #2 – Delayed Repetends

CALCULATOR: Casio *fx-300ES*

## INTRODUCTION:

A repeating decimal is a decimal that does not terminate (end), but eventually reaches a continuous pattern of digits. Examples are:

$$0.\overline{3} = 0.3333333\ldots \quad \text{and} \quad 25.41\overline{26} = 25.41262626\ldots$$

The portion of the decimal that repeats is called the repetend.

A delayed repetend is defined for the purposes of this activity as a repetend that does not begin its pattern immediately after the decimal point; rather, there is at least one digit after the decimal point that does *not* repeat.

All repeating decimals are members of the set of rational numbers. All rational numbers can be written as a ratio of two integers (an improper fraction).

## PROCEDURE:

Turn the calculator **[ON]**.

Make sure the *fx-300ES* is set up correctly for this activity by pressing the following key sequences:

**[SHIFT]-[SETUP] [1]** to enter Math Input/Output mode;

**[SHIFT]-[SETUP] [8] [1]** to enter Normal Display 1;

**[SHIFT]-[SETUP] [ ] [2]** to force fractions to display as improper fractions.

Type “1, 6” , then press **[=]**. The *fx-300ES* will display the fraction  $\frac{1}{6}$ .

Press the **[S D]** key. Notice that the fraction  $\frac{1}{6}$  is equivalent to the repeating

decimal  $0.16\overline{6666}\dots$ , or  $0.1\overline{6}$ . This decimal has a repetend delayed by one digit.

Let’s see if we can find a pattern amongst all repetends delayed by one digit.

Type “0.266666...” until the top line of the screen is filled with 6’s. Then press **[=]**. Notice that the *fx-300ES* interprets this as a repeating decimal, and outputs  $\frac{4}{15}$ .

*Exercise 1.* Use your fx-300ES, and any patterns you may observe, to fill in the following blanks. Your answers should be fractions in simplest form.

$$0.\overline{36} = \underline{\hspace{1cm}} \quad 0.\overline{46} = \underline{\hspace{1cm}} \quad 0.\overline{56} = \underline{\hspace{1cm}} \quad 0.\overline{76} = \underline{\hspace{1cm}} \quad 0.\overline{86} = \underline{\hspace{1cm}} \quad 0.\overline{96} = \underline{\hspace{1cm}}$$

Type “1.2343434...” until the repeating “34”s fill the top line of the screen. Then press [=]. The calculator outputs the improper fraction  $\frac{611}{495}$ . Since we initially typed in a number greater than 1, perhaps we would like to view this equivalent fraction as a whole number. To do so, press **[SHIFT]-[S D]** ( $a\frac{b}{c} \hat{=} \frac{d}{c}$ ). The output is  $1\frac{116}{495}$ .

*Exercise 2.* Use your fx-300ES, and any patterns you may observe, to fill in the following blanks. The answers in the two blanks should be (in simplest form) an improper fraction and a mixed number, respectively.

$$\begin{aligned} 2.\overline{345} &= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 3.\overline{456} &= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 4.\overline{567} &= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \\ 5.\overline{678} &= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 6.\overline{789} &= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \end{aligned}$$

From the results of Exercise 2, it seems that fractions with denominators of 495 can be expressed in the form of a repeating decimal. Test this idea by typing:

**[ $\frac{X}{X}$ ] [2][3][3] [ ] [4][9][5] [=] [S D]**

The resulting decimal is  $0.4707070\dots = 0.\overline{470}$ .

Now convert a mixed number to a decimal, by typing:

**[SHIFT]-[ $\frac{X}{X}$ ] ( $X\frac{X}{X}$ ) [1] [ ] [7] [ ] [3][0] [=] [S D]**

The resulting decimal is  $1.233333\dots = 1.\overline{23}$ .

(Note: You can force the fx-300ES to output fractions as mixed numbers by typing

**[SHIFT]-[SETUP] [ ] [1].** )

*Exercise 3.* Find a fraction or mixed number that is equivalent to a repeating decimal with a repetend delayed by two digits.

## SOLUTIONS TO EXERCISES:

Exercise 1. The correct fractions are as follows:

$$0.3\bar{6} = \boxed{\phantom{00}} \quad 0.4\bar{6} = \boxed{\phantom{00}} \quad 0.5\bar{6} = \boxed{\phantom{00}} \quad 0.7\bar{6} = \boxed{\phantom{00}} \quad 0.8\bar{6} = \boxed{\phantom{00}} \quad 0.9\bar{6} = \boxed{\phantom{00}}$$

The pattern can actually be seen more easily using a common denominator of 30:

$$\frac{11}{30}, \frac{14}{30}, \frac{17}{30}, \frac{23}{30}, \frac{26}{30}, \frac{29}{30}$$

$\frac{3}{30}$  is the difference from one fraction to the next, just as the decimals differ by 0.1.

(You may wonder why  $\frac{20}{30}$  seems to be missing from the middle of the sequence above; this is because  $\frac{20}{30} = \frac{2}{3} = 0.\bar{6}$ , which has no delayed repetend.)

Exercise 2. The correct fractions and mixed numbers are as follows:

$$\begin{array}{lll} 2.34\bar{5} = \boxed{\phantom{00}} = \boxed{\frac{\phantom{00}}{\phantom{00}}} & 3.45\bar{6} = \boxed{\phantom{00}} = \boxed{\frac{\phantom{00}}{\phantom{00}}} & 4.56\bar{7} = \boxed{\phantom{00}} = \boxed{\frac{\phantom{00}}{\phantom{00}}} \\ 5.67\bar{8} = \boxed{\phantom{00}} = \boxed{\frac{\phantom{00}}{\phantom{00}}} & 6.78\bar{9} = \boxed{\phantom{00}} = \boxed{\frac{\phantom{00}}{\phantom{00}}} & \end{array}$$

Again, the patterns can be seen more easily by using a common denominator of 495:

$$\frac{1161}{495}, \frac{1711}{495}, \frac{2261}{495}, \frac{2811}{495}, \frac{3361}{495} \quad (550 \text{ is the difference between numerators});$$

$$2 \frac{171}{55}, 3 \frac{226}{495}, 4 \frac{281}{495}, 5 \frac{336}{495}, 6 \frac{391}{495} \quad (55 \text{ is the difference between numerators}).$$

Exercise 3. The easiest way to find one such fraction or mixed number is simply to type in a decimal with the desired type of repetend. For example, type "3.40888888..." [=]. The calculator outputs the fraction  $\frac{767}{225}$  (which, by pressing **[SHIFT]-[S D]** ( $a \frac{b}{c} \hat{=} \frac{d}{c}$ ), can be seen to be equal to the mixed number  $3 \frac{92}{225}$ ).

In general, the fractions that will work are those that have denominators of 12, 36, 60, 75, 150, 180, 225, 300, 450, and 900 when written in simplest form.