

Investigating the Randomness of Numbers

Part I: Finding the Mean and Standard Deviation

If a sample is randomly selected from the population of numbers 1, 2, 3, ..., 1000, the *value* of the population that is expected to be reflected in the sample is the mean of the population. The expected value for the mean of a sample is the mean of the distribution of sample means, which is the mean of the population. Symbolically,

$$E(\bar{X}) = \mu(\bar{X}) = \mu$$

Since the set of numbers is a simple arithmetic progression, the sum of the numbers can be found using the process below.

$$(\text{first} + \text{last}) \times (\text{amount of numbers}) \div 2$$

NOTE TO THE TEACHER: The population is 500.5

To calculate the population standard deviation, students should use calculator lists.

1. Create the list of numbers from 1 to 250 in List 1 using the following process.
 - ❖ Cursor to the label “**List 1**” above the first entry for that list.
 - ❖ Select “**OPTN.**”
 - ❖ Select “**LIST**” (F1).
 - ❖ Select “**Seq**” (F5). Notice that “Seq(” appears at the left below the lists.
 - ❖ The parameters for entering a sequence are: variable name; expression using that variable, beginning value; ending value; increment. Enter the following: **X** (from the “**X, θ , T**” key; any variable would suffice); **X** (the expression is just the variable name); **1** (the beginning value); **250** (the ending value); **1** (the increment). In a moment, the sequence of numbers is created in List 1.
2. Find the squared differences between these numbers and the mean in List 2.
 - ❖ Cursor to the label for “**List 2.**”
 - ❖ Enter a left-parenthesis.
 - ❖ Select **OPTN.**
 - ❖ Select **LIST.**
 - ❖ Select “**List.**” Notice that the word “List” appears after the parenthesis.
 - ❖ Enter “**1**” to use List 1.
 - ❖ Enter the minus sign.
 - ❖ Enter the mean (**500.5**).
 - ❖ Enter a right-parenthesis.
 - ❖ Square the result (x^2). In a moment List 2 will be filled with the squared differences.
3. Repeat steps 1 and 2 above using lists 3 and 4, respectively, to first generate the list of numbers from 251 to 500 and then find the squared differences between those numbers and the mean.
4. Students need not repeat this process for the other numbers, because the results for the numbers 501 through 750 will be the same as those found in step 3 (in

- reverse), and the results for the numbers 751 through will be the same as those found in step 1 (again, in reverse). For example, the squared difference for the number 500 is $(500 - 500.5)^2$, which is 0.25, and the squared difference for 501 is $(510 - 500.5)^2$, which is also 0.25.
5. Sum List 2 and list 4
 - ❖ Select **"MENU."**
 - ❖ Select **"RUN."**
 - ❖ Select **OPTN.**
 - ❖ Select **LIST.**
 - ❖ Select **F6** to see more options.
 - ❖ Again select **F6** to see more options.
 - ❖ Select **"Sum" (F1).**
 - ❖ Select **OPTN.**
 - ❖ Select **LIST.**
 - ❖ Select **List.**
 - ❖ Enter the number of the list to be summed (**2**)
 - ❖ Enter the plus sign.
 - ❖ Repeat the above steps to sum **List 4.**
 6. Once the two lists have been summed, the result must be doubled to account for the squared differences of the other numbers (501 through 1000).
 7. This result is divided by 1000, the amount of numbers in the population, to determine the variance. The square root of the variance yields the standard deviation of the population.

NOTE TO THE TEACHER: The result should be 288.675.