Investigating the Randomness of Numbers
Part II: Testing the Means of Samples

NOTE TO THE TEACHER: The following five lists of numbers should be stored in a calculator so that the students can import the lists rather than typing the data into the lists. The first list was created using a small program for the *fx*-9750 calculator.

The remaining lists were concocted and staged so that they "appear" to be random if students only observe the first five entries shown when the lists first appear, which is why the students should not enter the data themselves. This was done to illustrate that a single technique for assessing randomness may not be sufficient. Even with both techniques studied in this and the following investigation, students should still use the human element to evaluate whether randomness is likely to exist.

1. List 1: 666, 665, 262, 558, 965, 225, 866, 697, 137, 102, 2261, 231, 562, 954, 172, 93, 884, 982, 145, 625, 670, 929, 107, 832, 471, 741, 600, 45, 746, 282, 303, 333, 101, 678, 757, 405, 671, 690, 825, 915, 870, 645, 808, 728, 450, 169, 398, 296, 346, 116
2. List 2: 25, 976, 1, 12, 1000, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 999, 998, 997, 996, 995, 994, 993, 992, 991, 990, 989, 988, 987, 986, 985, 984, 983, 982, 981, 980, 979, 978, 977
3. List 3: 476, 525, 500, 498, 511, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524
4. List 4: 191, 589, 798, 402, 995, 192, 193, 194, 195, 196, 197, 198, 199, 200, 392, 393, 394, 395, 396, 397, 398, 399, 592, 593, 594, 595, 596, 597, 598, 599, 600, 791, 792, 793, 794, 795, 796, 797, 799, 800, 991, 992, 993, 994, 996, 997, 998, 999, 1000
5. List 5: 385, 624, 746, 165, 983, 5, 25, 45, 65, 85, 105, 125, 145,185, 205, 225, 245, 265, 285, 305, 325, 346, 365, 405, 425, 445, 465, 485, 505, 525, 545, 565, 585, 605, 645, 665, 686, 705, 725, 765, 785, 805, 825, 845, 865, 885, 905, 925, 945, 965

NOTE TO THE TEACHER: These same lists will be used in Part III.

Students will now assess the decision regarding the null hypothesis that the mean of the sample equals the population mean. The population standard deviation is known, which rarely occurs in hypothesis testing (though it is often estimated). The population from which the numbers was selected certainly is not normal, but the test is of the mean of the sample, and the sample is sufficiently large for the sampling distribution of the means of such samples to be normally distributed. Therefore, the one-sample *z*-test can be used rather than the one-sample *t*-test. To perform the test, use the following procedure.

1. Select **MENU**.
2. Select **STAT**.
3. Select **TEST** (**F3**).
4. Select **Z** (**F1**).
5. Select the **1-sample** test (**F1**).
6. Data can come from a list or variable values. In this case, use a **list** (**F1**).
7. Test whether the means are **equal** (**F1**).
8. The population mean was calculated earlier to be **500.5**; enter and **EXE** to store.
9. The population standard deviation was calculated earlier to be **288.675**.
10. The list whose mean we are testing is **List 1** (**F1**).
11. The frequency of each number is **1** (**F1**).
12. **EXE** or **Calc** (**F1**).

The test statistic should be 0.46785. The p-value is 0.63988, because this is a two-tailed test. With alpha set at the customary value of 0.05, the decision would be to fail to reject the null hypothesis that the mean of the sample is the same as the mean of the population from which the sample was taken.

Now test the means of the samples found in Lists 2, 3, 4, and 5.

1. Selecting **EXIT** on the keyboard returns the students to the 1-sample z-test.
2. The only entry that must be changed is the list whose mean is to be tested. The list is changed by selecting the appropriate F-key (F1 through F6 for lists 1 through 6, respectively).

NOTE TO THE TEACHER: Students should find that the p-values for the means of Lists 2 and 3 are both 1, which would again mean that the null hypothesis would not be rejected. However, it is hoped that they will wonder whether such a value would likely occur in a truly random selection. They should also find that the p-value for List 4 is 0.019964 (the null hypothesis would be rejected at the 0.05 level), and the p-value for List 5 is 0.89205 (fail to reject).

Based on these results alone, students should conclude that lists 1, 2, 3, and 5 may be random while list 4 most likely is not. However, the mean of the sample is not the only aspect of the sample for which an expectation exists.