Investigating Negative Exponents

In earlier investigations we learned the three laws of exponents.

$$\text{Law I: } (x^a)(x^b) = x^{(a+b)}$$

$$\text{Law II: } x^a \div x^b = x^{(a-b)}$$

$$\text{Law III: } (x^a)^b = x^{ab}$$

When we first studied these laws of exponents, all exponents were positive, and $a > b$. In the investigation of an exponent of zero we saw that the three laws held even when exponents of zero were used, but the second law presented a special problem. The law had been defined when $a > b$ and expanded to include $a = b$. Suppose $a < b$. Then the second law would give an answer with a negative exponent. What is the meaning of a negative exponent? Before continuing, write in your own words what you think a negative exponent might be.

Now we are going to investigate the meaning of a negative exponent. Complete the following table using the power key (x^y) on the calculator to get the values in the "decimal" column. Remember that to enter a negative number you first enter the number without the negative sign and then press the "change sign" key (+/–) which is directly above the "7" key. To enter the values in the "fraction" column, read the value in the "decimal" column and write that number as a fraction. The last three fractions can and should be simplified. Finally, write the denominator of the fraction as a power of the same base found in column 1.

negative exponent	decimal	fraction	denom. as a power
10^{-1}			
10^{-2}			
10^{-3}			
2^{-1}			
2^{-2}			
2^{-3}			

How do you think 5^{-2} could be written using a positive exponent? _____

Evaluate 5^{-2} and your answer using the calculator. Are the answers the same? (They should be!)

Here is the definition of a negative exponent: If $x \neq 0$, $x^{-a} = 1/(x^a)$

Write in your own words why you think x cannot be zero in the above definition.

Another Way of Thinking About a Negative Exponent

Given the expression $2^2 \div 2^4$, the expression would be simplified using the second law of exponents as $2^{(2-4)}$, which is 2^{-2}. Using the definition of a negative exponent above, this should equal $1/(2^2)$, or 1/4. If the expression is evaluated by rewriting the exponents first, we have: $2^2 \div 2^4 = 4 \div 16 = 4/16 = 1/4$, which is the same answer.

Revisiting the Three Laws of Exponents

Do all three laws of exponents hold when negative exponents are used? In the table below, an expression involving two exponents is given in the first column. Use the laws of exponents to rewrite the expression using a single exponent, and enter your answers in the second column. Finally, evaluate the expressions in both columns using your calculator.

Column 1: expression	Column 2: single exponent	Evaluate expressions in column 1	Evaluate expressions in column 2
$(2^{-4})(2^0)$			
$(3^{-2})(3^2)$			
$5^3 \div 5^{-1}$			
$4^{-3} \div 4^{-2}$			
$(6^{-2})^{-2}$			
$(2^{-2})^3$			

You should see that the values in column 3 match those in column 4.

We have now completed our investigation of exponents. You have seen that the three laws of exponents hold whether the exponents are positive, negative or zero. Though we initially limited the numbers that the exponents could be, it turns out that there are no restrictions!