

Investigating the Meaning of an Exponent of Zero

In earlier investigations we learned the three laws of exponents.

$$\text{Law I: } (x^a)(x^b) = x^{(a+b)}$$

$$\text{Law II: } x^a \div x^b = x^{(a-b)}$$

$$\text{Law III: } (x^a)^b = x^{ab}$$

When we first studied these laws of exponents, all exponents were positive, and $a > b$. In which laws would it not matter whether a or b was greater? _____

Now we are going to investigate the meaning of an exponent of zero. Before beginning, write in your own words what you think an exponent of zero means.

Now complete the following table using the power key (x^y) on the calculator.

n	n^0	n	n^0	n	n^0
70		0.4		-3.07	
-2		-0.003		$\sqrt{2}$	
1		2.15		0	

All of the answers should not be the same. Which one(s) is/are different?

How would you complete this definition of a negative exponent?

If $n \neq$ _____, then $n^0 =$ _____

Now look at what you thought an exponent of zero might mean. Were you surprised by the results of this investigation?

Another Way of Thinking About an Exponent of Zero

The second law of exponents was developed when a was greater than b . What would happen if $a = b$?

What is $4^2 \div 4^2$? According to the second law of exponents, it should be $4^{(2-2)}$, which is 4^0 . According to the definition that you completed above, this should be 1.

$4^2 = 16$; $16 \div 16$ does equal 1, so the second law of exponents holds even if $a = b$.

Revisiting the Three Laws of Exponents

Do all three laws of exponents hold when exponents of zero are used? In the table below, an expression involving two exponents is given in the first column. Use the laws of exponents to rewrite the expression using a single exponent, and enter your answers in the second column. Finally, evaluate the expressions in both columns using your calculator.

Column 1: expression	Column 2: single exponent	Evaluate expressions in column 1	Evaluate expressions in column 2
$(2^7)(2^0)$			
$(3^0)(3^4)$			
$5^3 \div 5^0$			
$(6^2)^0$			
$(4^0)^6$			

You should see that the values in column 3 match those in column 4. Write a few sentences explaining in your own words why you think this makes sense. Keep in mind that any non-zero number raised to the zero power equals 1 and what happens when multiplying or dividing by 1 or raising 1 to a power.

Now you know that you can use the three laws of exponents even when exponents of zero are involved. Notice that the third row is the only example in which the second law of exponents is used and that the exponent of zero does not appear first. When we previously explored the second law of exponents, the exponent in the first position was greater than the exponent that followed. Earlier in this investigation we saw that the two exponents could be the same. However, we have not yet investigated what would happen if the first exponent is less than the second. If such is the case, the second law of exponents would give us a negative exponent in the answer. As of now you may not understand the meaning of a negative exponent. However, the last investigation into exponents will make the meaning clear.