

Advanced Algebra Activity: Investigating Logarithms – What is a Decibel Level?

CALCULATORS: Casio: *fx-9860G* or *fx-9860G Slim*

INTRODUCTION:

A logarithm is a function that is the inverse of exponentiation. The purpose of a logarithm is to be able to write exponents in a different way.

For example, the exponential equation $a=b^x$ can be re-written: $x=\log_b a$

The decibel is the unit that measures the intensity of a sound wave. It is named after the inventor of the telephone, Alexander Graham Bell.

This activity will use data from the following table:

Type of Sound	Intensity (W/m^2)	Decibel Level (dB)
Rustling leaf	$1 \cdot 10^{-11}$	10
Quiet library	$1 \cdot 10^{-8}$	40
Normal conversation	$1 \cdot 10^{-6}$	60
Vacuum cleaner	$1 \cdot 10^{-5}$	70
Truck (10 yards away)	$1 \cdot 10^{-3}$	90
Jet plane (50 yards away)	100	140

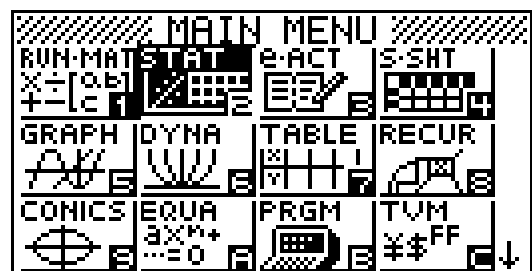
PROCEDURE:

From the Main Menu, select Statistics mode (mode #2).

Type the “Intensity” data from the table above into List 1. Since this data is mostly in scientific notation, you will need to use the C key. Here is an example of the keystrokes necessary to enter the first piece of “Intensity” data:

1cn11|

Finish entering the “Intensity” data. Remember to press | after each entry.



	List 1	List 2	List 3	List 4
SUB				
1	1E-11			
2				
3				
4				

	List 1	List 2	List 3	List 4
SUB				
4	1E-5			
5	1E-3			
6	100			
7				

TOOL EDIT DEL DELA INS D

Press B repeatedly until the cursor is highlighting the space just beneath "List 1".

Input the title "INTENSITY" by pressing La to Alpha-Lock the keypad, then pressing the keys with the letters "I", "N", "T", etc. printed above them.

Notice that list titles can only accept names up to 8 characters (the "Y" in "INTENSITY" does not display).

	List 1	List 2	List 3	List 4
SUB				
1	1E-11			
2	1E-8			
3	1E-6			
4	1E-5			
INTENSITÄ				

Press I when you're done to store the title.

Press \$ to move over to List 2.

Exercise 1. Type the "Decibel" data (again from the table on Page 1) into List 2, and give List 2 the title "DECIBEL".

Now we wish to investigate the relationship between Intensity and Decibel Level.

Let's begin by looking at a scatterplot of the points described by the two lists.

Press q (GRPH), then q (GPH1).

Unfortunately, the graph is not very useful. The only point we can see clearly is (100, 140) from the last row of our data table. The rest have intensities so small (0.001, 0.00001, etc.) that the points get hidden behind the y-axis.



Press Lp (SETUP), then scroll down until you reach the "Axes" option. Press W to toggle the axes Off in the graph window.

Background	: None	↑
Sketch Line	: Norm	
Angle	: Rad	
Complex Mode	: Real	
Coord	: On	
Grid	: Off	
Axes	: Off	↓
	On Off	

Press d. Notice that you do not return to the graph, but to the main Statistics screen. This is because you have changed settings.

Exercise 2. With the axes turned off, graph the scatterplot again.

Press q (CALC) to bring up a menu of statistical regressions. These can be used to calculate the likelihood of certain relationships between two variables.

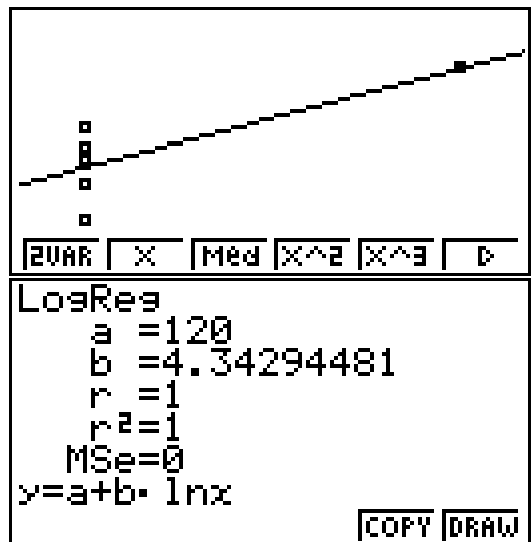
```
LinearReg
  a =0.86000607
  b =53.9997537
  r =0.78969301
  r²=0.62361506
  MSe=929.984448
y=ax+b
                [COPY] [DRAW]
```

Press W (x) to perform a linear regression.

The correlation coefficient r is fairly close to +1, which usually means there is a positive linear relationship. But is this really the case? We need to look at the graph.

Press u (DRAW) to graph the regression line.

Clearly this line does not describe the Intensity/Decibel data very well. Most of the data points appear vertical (near or on the now-invisible y-axis), rather than the diagonal that the line suggests. Let's try a different regression test:



Press u (▷) to reveal more regression options. Then press W (Log) to perform a logarithmic regression test.

Now r is equal to +1! This is indisputable evidence that there is a logarithmic relationship between Intensity and Decibel Level. In fact, the equation is shown at the bottom of the screen:

$y = a + b \cdot \ln x$...or, since $a = 120$ and $b = 4.34294481$,
$y = 120 + 4.34294481 \cdot \ln x$	

Press u (DRAW) to graph the logarithmic regression curve.

Press Lr (Sketch), then q (CIs) to clear the diagonal regression line.

CONCLUSION:



The actual equation relating sound intensity and decibel level involves base-10 logarithms, and can be derived from the regression equation in this activity using change-of-base:

$$\begin{aligned}
 y &= 120 + 4.34294481 \times \ln x \\
 &= 120 + 4.34294481 \times \frac{\log x}{\log e} \\
 &= 120 + 4.34294481 \times \frac{\log x}{0.434294481} \\
 y &= 120 + 10 \log x
 \end{aligned}$$

In other words, *Decibel Level* = $120 + 10 \log(\textit{Intensity})$.

This relationship is why decibels are considered a measure of relative intensity.

For more information on this logarithmic relationship, visit:

http://physics.mtsu.edu/~wmr/log_3.htm

<http://www.ndt-ed.org/GeneralResources/decibel/decibel.htm>

http://www.ihr.mrc.ac.uk/research/intensity_decibels.php

http://www.sciencebuddies.org/mentoring/project_ideas/HumBeh_p007.shtml

<http://library.thinkquest.org/19537/Physics8.html>

SOLUTIONS TO EXERCISES:

Exercise 1. To enter the data, use the following keystrokes:

101

	List 1	List 2	List 3	List 4
SUB	INTEN			
4	1E-5	70		
5	1E-3	90		
6	100	140		
7				

TOP EDIT DEL DEL INS D

40I
60I
70I
90I
140I

Then, press B enough times so that the cursor is directly below the text “List 2”, and type:

La (A-LOCK)

hjGjgjbI

D E C I B E L

	List 1	List 2	List 3	List 4
SUB	INTEN	DECIBE		
1	1E-11	10		
2	1E-8	40		
3	1E-6	60		
4	1E-5	70		

10

TOOL EDIT DEL DELA INS D

Exercise 2. The keystrokes necessary are simply q (GRPH), then q (GPH1).

