

Matrix Rotations

Lesson Plan Level: Geometry; Algebra II

Graphing Calculate: FX-9860G Plus; FX-9860 G Slim

Objective: The student will be able to:

1. Use the RUN-MAT Menu of a graphing calculator to find the new coordinates of a polygon that is rotated about the origin point through the angles of -90° , 90° , and 180° , and
2. Verify the results algebraically.

Engage

Review the terminology related to coordinate axes, especially clockwise and counterclockwise movement. Discuss with students the meaning of a translation and demonstrate a simple rotation without coordinates. Discuss how rotations are used in graphic arts and design.

Explore

Using the calculator, model using the RUN-MAT Menu of the graphing calculator and matrix operations.

Explain:

Students will be able to explain the changes in the coordinates of a polygon for each of the given rotations.

Evaluate:

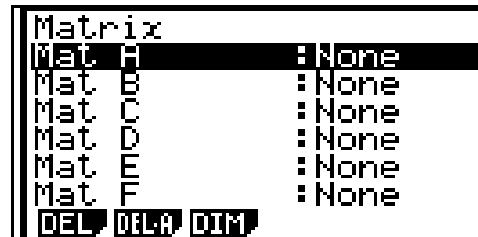
The student will be able to demonstrate their results using the calculator and an over head or LCD projector.

Extensions:

The students will be able to create a simple graphic design using rotations of lines and polygons using a geometry program.

Calculator Notes: Operations with Matrices

To set up the calculator and access the matrix editor in the RUN-MAT Menu using the linear mode, press **SHIFT** **MENU** **F2** **EXE** **F1**. The screen will look like the one at the right. Notice the soft key menus at the bottom of the screen.



These are the uses for the soft keys.

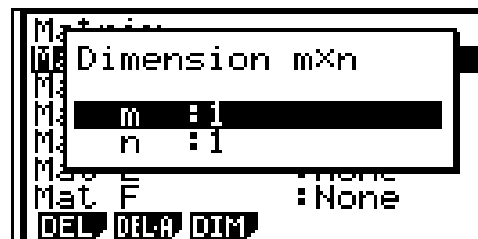
DEL - This is used to delete the contents of the highlighted matrix.

DEL-A - This is used to delete the contents of all the matrices.

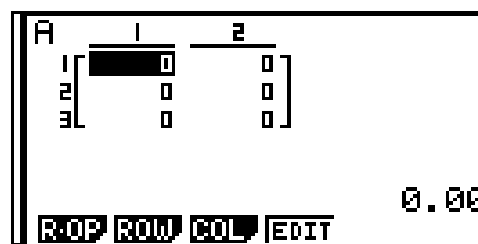
DIM - This is one way to set up the dimensions of the matrix.

Press the **▶** key and the screen will change as shown at the right. In this screen, m represents the number of rows and n represents the number of columns. For this activity, you will want to enter

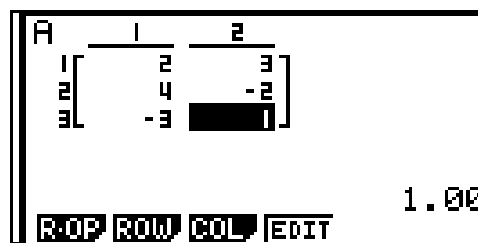
3 **EXE** **2** **EXE** **EXE**.



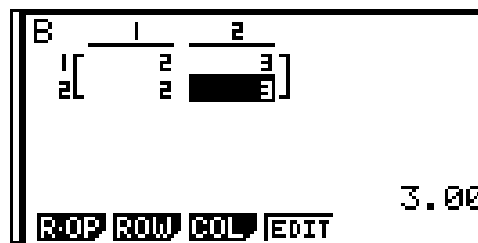
The screen will appear as shown at the right. To enter the values into the matrix, input the value that corresponds to the highlighted cell and press **EXE**.



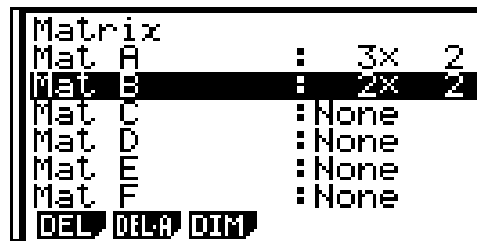
Inputting the vertices of (2, 3), (4, -2), and (-3, 1) will result in the screen at the right. Press **EXIT**.



Press **▼** **▶** **2** **EXE** **2** **EXE** **EXE** and enter the values of **2** **EXE** **3** **EXE** **2** **EXE** **3** **EXE**.



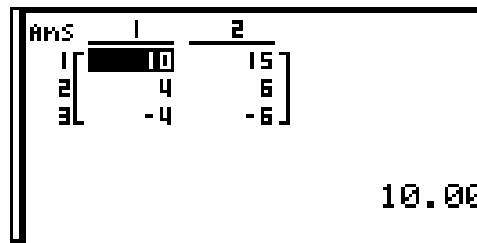
The calculator now has two matrices in memory. The first is MAT A and the second is MAT B as shown at the right. Press **EXIT**.



To multiply two matrices, press **OPTN** **F2** to the matrix soft keys.



Now press **F1** **ALPHA** **X,θ,T** **×** **F1** **ALPHA** **log** **EXE** to find the product. The screen should resemble the one on the right. Pressing **AC/ON** twice will return the calculator to the main screen for matrix operations.



Pressing **EXIT** three times will return you to the main RUN-MAT Menu. Press **F1** to get back to the matrix editor. Highlight the matrix you want to change and press **EXE**. Enter the new matrix and repeat the process.

Student Activity Sheet - Matrix Rotations

Name: _____

Date: _____

Objective: Given the coordinates of a polygon, the student will:

1. Use a graphing calculator to create a matrix for the coordinates of the polygon,
2. Use a graphing calculator to create matrices to reflect the polygon across the x-axis, y-axis, and the line $y = x$,
3. Find the new coordinates of the vertices for the polygon for each reflection, and
4. Verify this with the graphing calculator using a GEOM program.

Introduction:

One of the many uses of reflecting points is in the use of graphic design and building construction. These uses can be seen in fabric patterns, architecture, and animation. There are also uses for translating radio waves for cell phones and satellites.

Materials:

- a. Casio FX-9860G+ Graphing Calculator
- b. Student Answer Sheet

Formulas:

$$a. \begin{bmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ x'_3 & y'_3 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ - Counterclockwise Rotation}$$

$$b. \begin{bmatrix} x'_1 & y'_1 \\ x'_2 & y'_2 \\ x'_3 & y'_3 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \text{ - Clockwise Rotation}$$

Procedure:

1. Use the calculator to find the new coordinates for Triangle ABC and fill in the table.
2. Use the calculator to find the new coordinates for Parallelogram ABCD and fill in the table.
3. Verify your answers algebraically on a separate sheet of paper.
4. Answer the questions concerning your findings.

Table 1:

Triangle ABC		Triangle A'B'C'			
			-90°	90°	180°
Point A	(5, 2)	Point A'			
Point B	(-1, -3)	Point B'			
Point C	(2, -2)	Point C'			

Table 2:

Parallelogram ABCD		Parallelogram A'B'C'D'			
			-90°	90°	180°
Point A	(6, 2)	Point A'			
Point B	(4, -2)	Point B'			
Point C	(-4, -2)	Point C'			
Point D	(-2, 2)	Point D'			

Questions:

1. What change takes place when a figure is rotated -90° ? _____

2. What change takes place when a figure is rotated 90° ? _____

3. What change takes place when a figure is rotated 180° ? _____

4. Did you get the same results using algebra? _____
5. If a triangle whose vertices are $(-12, 10)$, $(6, -8)$ and $(-3, 4)$ is rotated -90° , what would be the coordinates of the new triangle? _____
6. If a rectangle whose vertices are $(-5, 4)$, $(-5, -2)$, $(3, -2)$ and $(3, 4)$ is rotated 90° , what would be the coordinates of the new rectangle? _____
7. In your own words, describe how to find the new coordinates for a rotation of 180° .

Solutions:

Table 1:

Triangle ABC		Triangle A'B'C'			
			-90°	90°	180°
Point A	(5, 2)	Point A'	(-2, 5)	(2, -5)	(-5, -2)
Point B	(-1, -3)	Point B'	(3, -1)	(-3, 1)	(1, 3)
Point C	(2, -2)	Point C'	(2, 2)	(-2, -2)	(-2, 2)

Table 2:

Parallelogram ABCD		Parallelogram A'B'C'D'			
			-90°	90°	180°
Point A	(6, 2)	Point A'	(-2, 6)	(2, -6)	(-6, -2)
Point B	(4, -2)	Point B'	(2, 4)	(-2, -4)	(-4, 2)
Point C	(-4, -2)	Point C'	(2, -4)	(-2, 4)	(4, 2)
Point D	(-2, 2)	Point D'	(-2, -2)	(2, 2)	(2, -2)

Questions:

1. The coordinates are reversed and the sign of the new first coordinate changes to the opposite.
2. The coordinates are reversed and the sign of the new second coordinate changes to the opposite.
3. The signs of the coordinates change to their opposites.
4. Yes
5. (-10, -12) (8, 6) (-4, -3)
6. (4, 5) (-2, 5) (-2, -3) (4, -3)
7. Answers will vary.