Algebra/Spreadsheet Activity:
Finding Pythagorean Triples, part 3

CALCULATORS: Casio: *fx-9860G*

INTRODUCTION:

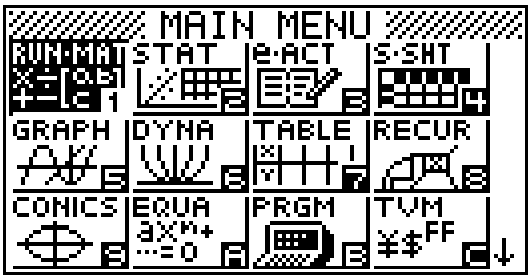
A Pythagorean Triple is a sequence of three positive integers (*a, b, c*) which satisfy the Pythagorean Theorem ($a^2 + b^2 = c^2$).

In part 2 of this Activity, the Spreadsheet mode of the *fx*-9860G was used to generate Pythagorean triples using ascending odd values of *a* (3, 5, 7, ...) .

This third and final part of the Pythagorean Triples activity will use the Spreadsheet mode to generate **any** Pythagorean triple (including scalar multiples of other triples).

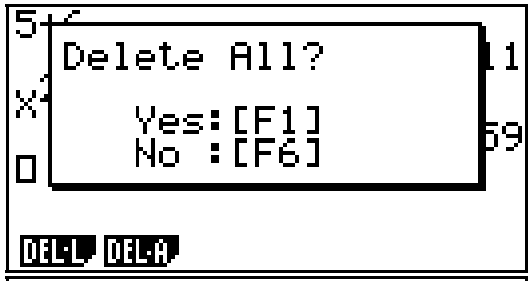
PROCEDURE:

From the Main Menu, select the Run-Matrix mode (mode #1).



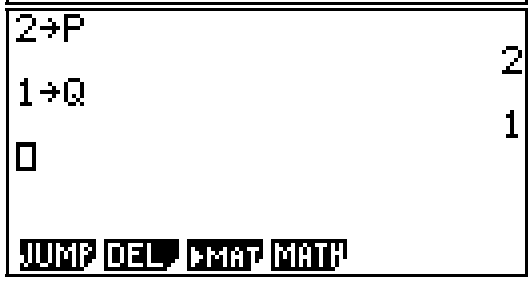
Press **[SHIFT]-[MENU]** to enter the **SET UP** menu, and press **[F1]** to make sure you are in **Math** input mode. Press **[EXIT]** to return to the data entry area.

Press **[F2]** (DEL), then **[F2]** (DEL-A). Choose “Yes” by pressing **[F1]**. This will clear the screen in **Math** entry mode.



Store “2” and “1” as the values of variables P and Q, by typing:

[2] [ ] [ALPHA]-[4] [EXE]
[1] [ ] [ALPHA]-[5] [EXE]



Evaluate the expressions $P^2 - Q^2$ and $2PQ$:

[ALPHA]-[4] [x^2] [–] [ALPHA]-[5] [x^2] [EXE]

[2] [ALPHA]-[4] [ALPHA]-[5] [EXE]

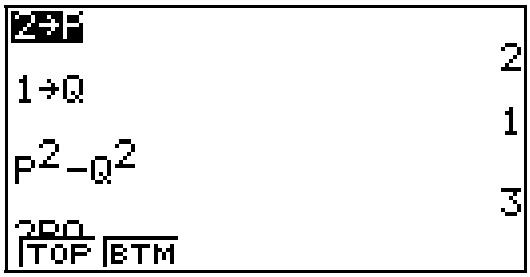
If entered correctly, the two expressions yield the values “3” and “4” respectively.

<u>Exercise 1.</u> Use the fx-9860G to evaluate the expression P^2+Q^2*, where P=2 and Q=1.*

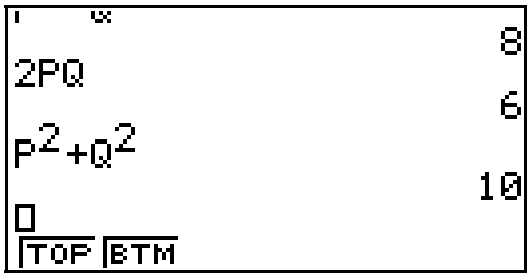
When P=2 and Q=1, the three expressions $P^2 - Q^2$, $2PQ$, and P^2+Q^2 generate a Pythagorean triple! Let’s investigate whether this is true for other values of P and Q:

Press **[F1]** (JUMP), then **[F1]** (TOP) to navigate to the first entered command (where “2” was stored as the value of variable P).

Press **[ ]** to begin editing the first command. Press **[ ] [DEL] [3] [EXE]** to change the value of P to 3.



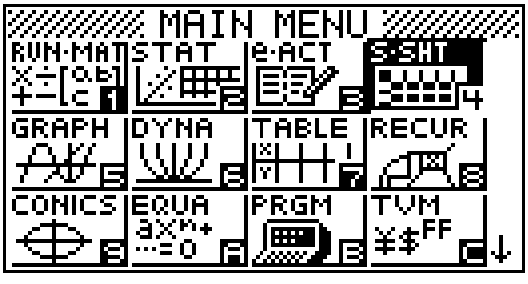
Notice that the cursor scrolls back to the bottom of the command list, changing any commands that were dependent upon the edited command. As a result, the three expressions now show a new Pythagorean triple: (6, 8, 10). This triple is a scalar multiple of the first triple, (3, 4, 5).



<u>Exercise 2.</u> Determine the Pythagorean triple generated by the values P=3 and Q=2.

Repeatedly editing the values of P and Q in Run-Matrix mode can be tedious. It is much easier to use Spreadsheet mode to experiment with P and Q:

Press **[MENU]**, and select the Spreadsheet mode (mode #4).

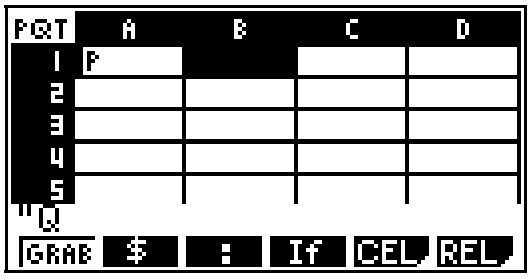


Press **[F1]** (FILE), then **[F1]** (NEW) to create a new named spreadsheet. Enter “PQTRIPS” as the name of this spreadsheet, then press **[EXE]**.

Press **[SHIFT]-[ALPHA]** to activate **Alpha-Lock**. Then press **[EXP] [4] [EXE]**.
(Notice that all keys now output the red alphabetic characters written above the keys. This lasts until **[EXE]** is pressed. Also note that no closing quote is necessary.)

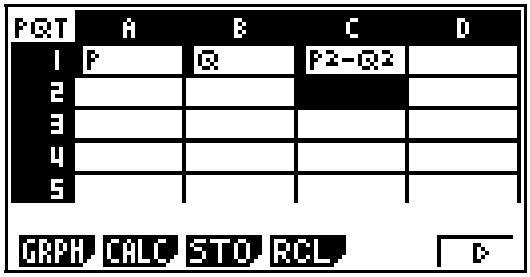
Press **[ ] [ ]** to navigate to cell B1. Create the letter Q in this cell:

[SHIFT]-[ALPHA] [EXP] [5] [EXE]



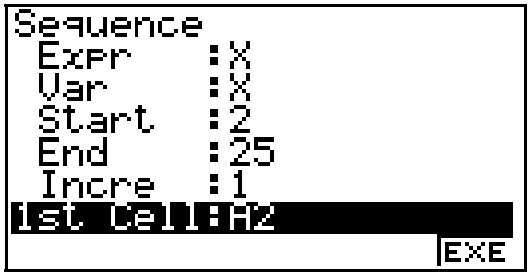
Navigate to cell C1 and enter the expression $P^2 - Q^2$. (Do not use **Alpha-Lock** here.)

[ALPHA]-[EXP] [ALPHA]-[4] [x^2] [–] [ALPHA]-[5] [x^2] [EXE]



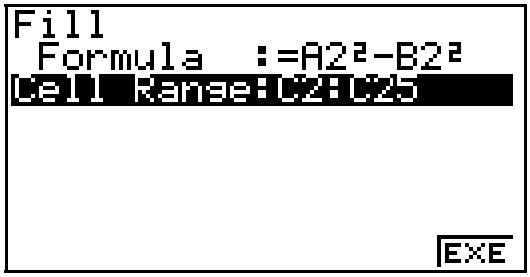
Use the same technique to enter the expressions $2PQ$ and $P^2 + Q^2$ in cells D1 and E1, respectively.

Press **[F2]** (EDIT) followed by **[F5]** (SEQ) to reach the sequence entry screen. Fill in the values as shown to the right, then press **[F6]** (EXE). This fills the next 24 rows of column A with the integers 2 through 25. (Press **[ ]** four times to show column A.)



Use similar syntax to fill rows 2 through 25 of column B with the integers from 1 through 24. (The values in the Sequence screen should be X, X, 1, 24, 1, B2.)

Navigate to cell C2. Press **[F6]** (▷), then **[F1]** (FILL). Input the formula “$=A2^2 + B2^2$”, and the cell range “C2:C25”.

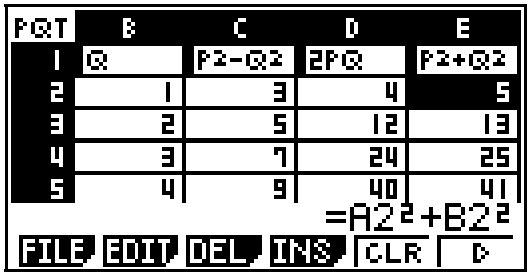


[SHIFT]-[.] [ALPHA]-[X,q,T] [2] [x^2]
[–] [ALPHA]-[log] [2] [x^2] [EXE]

[ ][ ][ ][ ][ ][ ] [5] [EXE]

Then press **[F6]** (EXE).

Beginning with **[F1]** (FILL), use the same technique to fill cell range D2:D25 with the formula "=2(A2)(B2)", and cell range E2:E25 with the formula "$=A2^2 - B2^2$".

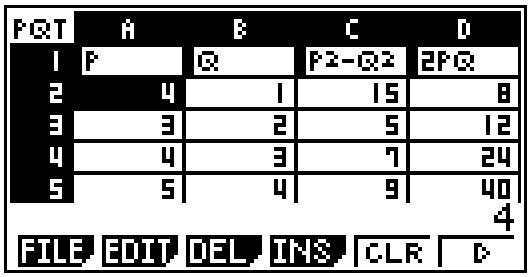


Navigate to cell F2. (This will make column C the left-most column visible.)

Columns C, D, and E now show the same Pythagorean triples that were generated in Activity 2, except that in this case, the triples were generated by the seed values P and Q, rather than using a formula dependent upon the choice of *a*.

Currently, P and Q are chosen so that P=Q+1. However, **any** P and Q can be used, as long as P and Q are positive integers with P > Q.

Navigate to cell A2. Press **[4] [EXE].** The values in cells C2, D2, and E2 change to reflect this new value of P, generating the new Pythagorean triple (8, 15, 17).



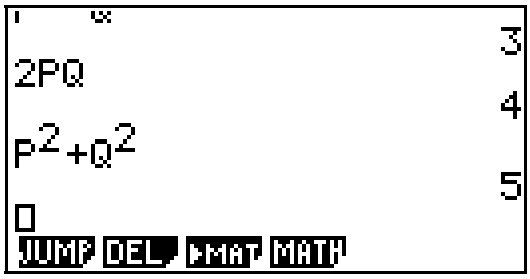
<u>Exercise 3.</u> Use the Sequence screen to place the integers 4 through 27 in cells A2 through A25. Use the resulting spreadsheet to find the missing number in each of these Pythagorean triples:

(20, 21, _____) (45, 108, _____) (81, 360, _____) (111, _____, 689)

SOLUTIONS TO EXERCISES.

<u>Exercise 1.</u>

At this point in the activity, the calculator already has the proper values of P and Q stored. Input the expression P^2+Q^2 *using these keystrokes:*



[ALPHA]-[4] [x^2 ***] [–] [ALPHA]-[5] [*** x^2 ***] [EXE]***

The correct value of the expression is 5 .

Exercise 2.

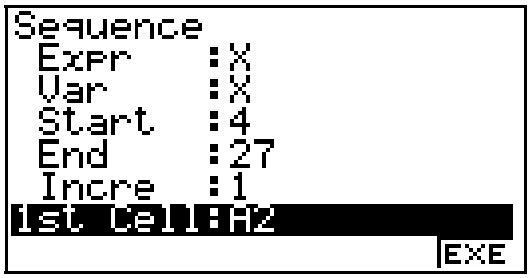
Since the value of P is already set to 3, only the following commands are required:

[F1]** (TOP) **[ ][ ] [ ] [ ] [DEL] [2] [EXE]

This generates the Pythagorean triple (5, 12, 13) .

Exercise 3.

*Press **[EXIT]** as necessary, until **[F2]** (EDIT) becomes a menu option. Then press **[F2]** (EDIT), **[F5]** (SEQ), and input the values X, X, 4, 27, 1, A2.*



The missing values are:

*(20, 21, __**29**__)* *(45, 108, __**117**__)*

*(81, 360, __**369**__)* *(111, __**680**__ , 689)*