CLEMSON MIDDLE SCHOOL MATHEMATICS PROJECT
UNIT 3: PROPORTIONAL REASONING

PROBLEM 1: HOW MUCH TRASH?

A few years ago, "Newsday" collected and analyzed the trash disposed of by a family of 6 named Sigmann. Their study took place over a four-week period.

A. Consider the data given by "Newsday." Enter the data into your calculator.

ITEM	**WEIGHT**
Newspapers	42 lbs. 11 ozs.
Food	42 lbs. 4 ozs.
Paper	32 lbs. 7 ozs.
Plastic	19 lbs. 11 ozs.
Other	29 lbs.

B. Calculate the total amount of trash produced by the Sigmanns. Determine the percent of trash each item represents and display the results in a pie graph.

C. Holbrook, the town in which the Sigmanns lived, had a population of 25,273. If the Sigmanns were representative of the entire population, determine how much trash the town produced in four weeks. How much trash would the town have produced in a year?

D. At these rates, how much trash would the United States produce in a year?

E. Create a graph that relates the amount of trash each person produces and the number of years the person is alive. Use your graph to determine how much trash you would have produced by the time you reach your 80^{th} birthday. What assumptions are you making when you compute this? Do you think these assumptions are reasonable? Explain.

F. Suppose you reduce your total trash by 10% and recycle 40% of what you still produce. At this rate, by how much would you reduce your trash during the course of a year?

G. If everyone in the U.S. also reduced the total amount by 10% and recycled 40% of what was still produced, by how much would the trash be reduced during the course of a year?

H. Write a short essay describing how these reductions might be accomplished.

MATERIALS:

Casio *Algebra FX 2.0* Graphing Calculator

EXTENSION:

1) As a class project, collect and analyze your own trash over a two-week period. Then using your own data, first determine any significant changes that you think may have occurred since "Newsday" collected their data. Then, answer the questions above, but use your own town, not the Sigmanns' town. Finally, write your own questions that might help others become aware of the trash that they produce.
2) Prepare a presentation for your town or county council to convince them to adopt a recycling program for your town or county.

REFERENCE: Understanding Rational Numbers and Proportions, NCTM Addenda Series, 1996, pp. 32-48.

ONE SOLUTION TO PROBLEM 1: HOW MUCH TRASH?

A. Enter the data found by "Newsday" into a list on your calculator.

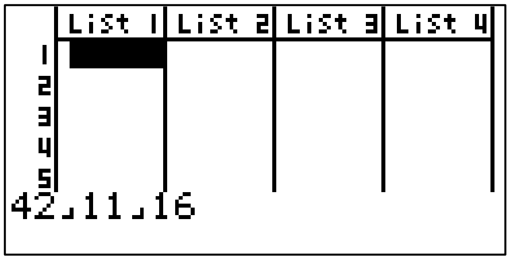
Because there are 16 ounces in a pound, all of the measures with ounces can easily be entered in pounds by using the fraction capabilities of the Casio *Algebra FX 2.0*. For example, 42 pounds, 11 ounces can be entered as $42\frac{11}{16}$. To do so, simply press the fraction key after the whole number and again after the numerator. Note that you do not have to reduce the fractions to enter them, although you may certainly choose to do so.

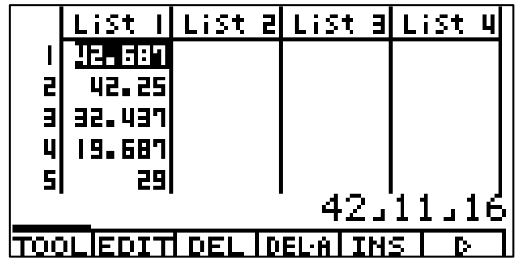
ITEM	**WEIGHT**	**WEIGHT**
Newspapers	42 lbs. 11 ozs.	$42\frac{11}{16}$
Food	42 lbs. 4 ozs.	$42\frac{4}{16}$
Paper	32 lbs. 7 ozs.	$32\frac{7}{16}$
Plastic	19 lbs. 11 ozs.	$19\frac{11}{16}$
Other	29 lbs.	29

To enter the data into the calculator, from the MAIN MENU, choose STAT. (If necessary, press [ESC] to get the lists screen.) Note that you can enter the data in fractional form, however, in the list, the data will appear in decimal form.

- Before entering the data, clear any data from List 1. To do so, with the cursor any place within list 1, press [F6] until DEL-A (delete all) appears at the bottom of the screen. Then press [F4] to delete the existing data followed by [EXE] to clear the list.
- Type in each number, separating the whole number from the fraction and the numerator of the fraction from the denominator by using the [a b/c] key. For

example, to enter $42\frac{11}{16}$, type 42, [a b/c], 11, [a b/c], 16, then [EXE]. See below left. After you have completed each number, be sure to press [EXE]. The top of the list after all entries have been made is shown below right. Note both decimal and fraction forms for the top entry.

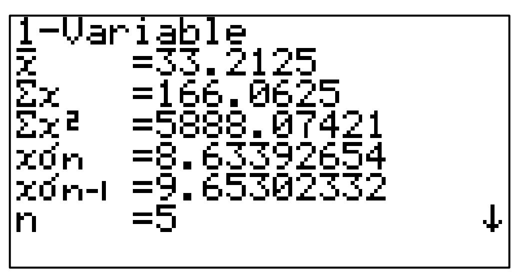




B. Calculate the total amount of trash produced by the Sigmanns. Determine the percent of trash each item represents and draw a pie chart to represent the data.

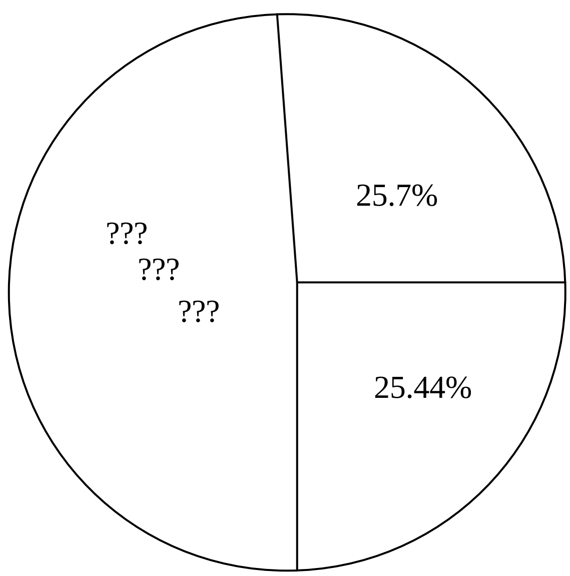
The one-variable statistics that the Casio *Algebra FX 2.0* calculates include the sum of the data. After the data have been entered,

- If necessary, press [F6] for more options so that CALC is shown on the screen. Then press [F2] to access the CALC menu.
- Press [4] for Set. Make sure the 1VarXList is set at List 1 (if not, with 1VarX highlighted, press [F1] then [1]) and the frequency is set to 1 (if necessary, with 1Var Freq highlighted, press [F1]) so that each value is used one time. See below left. Then press [ESC] to return to the primary statistics screen.
- Press [F2] for calculate and [1] for one-variable statistics. See below right.



The total amount of trash, represented by the sum, is just over 166 pounds. (? *x* represents the sum of the data in the list.)

- To figure the percentages for the pie chart, press [MENU] to return to the Main Menu screen. Choose the RUN-MAT.
- Enter each weight using the fraction key, [a b/c] , as before, and divide each weight by the total ? x from the 1-variable statistics (166).
- The results are 25.7%, 25.44%, 19.53%, 11.85%, 17.46% respectively. This is an excellent opportunity to discuss approximating parts of the whole with the students.
- Have students draw a circle and partition the appropriate amounts for each percentage. Many of your students will need guidance with how to do this. For example you may have them notice that the first two percents are about half of the whole pie chart. See possible partition below. Guide your students in completing the pie chart with the remaining percentages. While exactness is not the goal, good approximating is, so, for example, be sure no student has 11.85% partitioned in an area that is larger than any of the others.



C. Holbrook, the town in which the Sigmanns lived, had a population of 25,273. If the Sigmanns were representative of the entire population, determine how much trash the town produced in four weeks. How much trash would the town have produced in a year?

If we assume that the Sigmanns would have continued to produce trash at the same rate, we could solve the following proportion to determine how much trash they would produce in a year.

$$\frac{166.062\,pounds}{4weeks} ? \frac{a}{52weeks}$$

To solve this for a, we can multiply both sides by $4weeks ? 52weeks$. Reduce (cancel) out forms of 1 on both sides, and we have $166.062\,pounds ? 52weeks ? a ? 4weeks$.

We can then divide both sides by $4weeks$, obtaining

$$\frac{166.062\,pounds ? 52weeks}{4weeks} ? \frac{a ? 4weeks}{4weeks}.$$

Doing the arithmetic from the RUN menu we get $a ? 2160\,pounds$.

Because solving proportions is such a common operation, we sometimes combine these steps. Notice that our steps are equivalent to cross-multiplying and then dividing. In other words, from the proportion we can proceed directly to

$$a ? \frac{166.062\,pounds ? 52weeks}{4weeks} ? 2160\,pounds.$$

We still, however, have not answered the question. We can use another proportion to determine how much trash the entire town would produce in a year, assuming again that the rate remains the same. Recall that there were 6 people in the Sigmann family. Again after setting up the proportion, we can do the arithmetic from the RUN menu.

$$\frac{6\,people}{2160\,pounds} ? \frac{25273\,people}{z}$$

Solving with our simplified technique, we find that

$$z ? \frac{2160\,pounds ? 25273\,people}{6\,people} ? 9{,}000{,}000\,pounds.$$

D. At these rates, how much trash would the United States produce in a year?

Once again, we can solve a proportion. We will assume the population of the United States is approximately 280,000,000. The left side of the proportion shown below is again based on the Sigmanns.

$$\frac{6\,people}{2160\,pounds} ? \frac{280{,}000{,}000\,people}{q}.$$

Solving, we find that $q ? \frac{2160\,pounds ? 280{,}000{,}000\,people}{6\,people} ? 1 ? 10^{11}\,pounds$. This represents 100,000,000,000 (100 billion) pounds of trash.

E. Create a graph that relates the amount of trash each person produces and the number of years the person is alive. Use your graph to determine how much trash you would have produced by the time you reach your 80th birthday. What assumptions are you making when you compute this? Do you think these assumptions are reasonable? Explain.

Before creating the graph, we must realize that we are assuming that the rate of trash creation determined by the four weeks of research is maintained throughout a person's lifetime. This may not be realistic, but we should ask ourselves if we have any evidence to assume that the rates will change.

First, let's determine how much trash a single person is producing each year, based on our evidence. Again, we can use proportional reasoning.

$$\frac{6\,people}{2160\,pounds} ? \frac{1\,people}{z}$$

Solving, we find that a person (on average) produces about 360 pounds of trash per year.

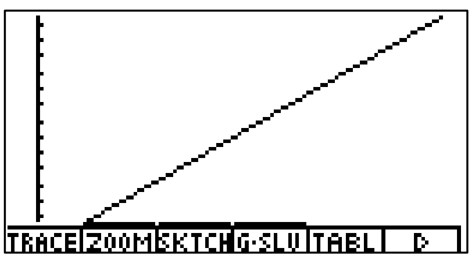
Before we create our graph, let's assign variables.

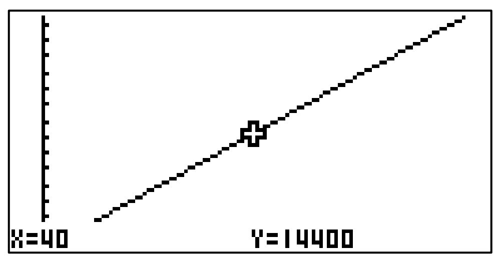
?? Let x represent the age of the person in years.

?? Let y represent the amount of trash the person has produced.

Assuming a constant rate of 360 pounds per year, we can then relate our two variables by the formula $y ? 360x$. To create a graph of this relationship, from the MAIN MENU, choose GRPH-TBL. Then,

- Delete any functions that are present by highlighting them and pressing [F2] followed by [EXE].
- With Y1 highlighted, type in 360x, using the [X,?,T] for the x, then press [EXE].
- Press [SHIFT] [OPTN] to set the viewing window. The following values are possibilities. Press [EXE] after each entry. Xmin: -5; max: 85; scl; 10. Leave dot set as is, using the down arrow on the disc to bypass. Now set Ymin: 0; max: 80 ? 360 (you might as well let the calculator calculate!); scl: 2000.
- After all of these entries have been made, press [ESC] and [F5] to view the graph. See below left.
- To trace through the graph, press [F1] and use the arrow keys on the disc to move the cursor along the graph. See below right for a point that tells us that by the time you turn 40 you will have produced 14,400 pounds of trash.





Students should note that the graph goes through (0, 0). They should understand that this means that at birth they have yet to produce any trash. They should also note that the graph is linear, and goes up 360 pounds for every one year it moves to the right. With experience, students should recognize that graphs of proportional relations (examples of direct variation) always go through the origin and are linear.

F. Suppose you reduce your total trash by 10% and recycle 40% of what you still produce. At this rate, by how much would you reduce your trash during the course of a year?

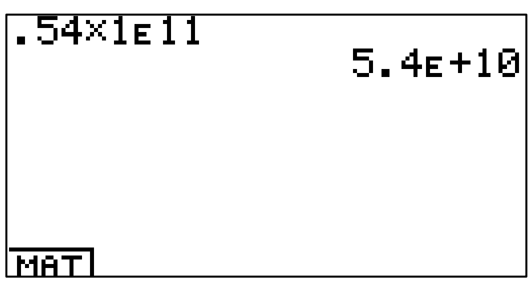
Without reduction, we have found that, based on the rates presented in this problem, the amount of trash a person produces each year is approximately 360 pounds. One way to compute the results from the suggested reductions is to use the RUN-MAT mode to find 360 ? 10% ? 360 , obtaining 324, and then finding 324 ? 40% ? 324 , obtaining 194.4 pounds.

A simpler way is to recognize that if we reduce the trash by 10%, we still produce 90%. If we then recycle 40%, we then continue to produce 60% of the 90%. In other words, all we need do is find .60 ? (.90 ? 360) , which once again gives us 194.4 pounds. Using the associative property, we can determine that by making the suggested reductions, we will then produce only 54% as much trash as our original amount, a reduction of 46%.

G. If everyone in the U.S. also reduced the total amount by 10% and recycled 40% of what was still produced, by how much would the trash be reduced during the course of a year?

Using the same logic, we simply need to compute 54% of 100,000,000,000. On the calculator in the from the MAIN MENU choose RUN-MAT. We need to enter 100 billion in scientific notation. To do so,

- Press .54, the multiplication symbol, 1, EXP , 11, and EXE . See below.



The result is $5.4 ? 10^{10}$, or 54,000,000,000 (54 billion) pounds. Although this is still a huge number, it does represent a reduction of 46 billion pounds, a very significant amount.

H. Write a short essay describing how these reductions might be accomplished.

Answers will vary. If desired, this question can be turned into a research project.

PROBLEM 2: GET THE PICTURE?

A. From left to right, the five people in the picture taken by a Casio 8000 digital camera, are Jeff, Gwen, Jill, Joycelyn, and Ryan, graduate students at Clemson University. (After the copying process, you can't tell how good-looking these people really are!). Gwen is 5' 3" tall. Use this information to determine the heights of the other four people. What assumption(s) must you make?
B. A desk is 80 centimeters tall. How tall would it be in the picture? What about a filing cabinet that actually stands 134 centimeters?
C. Suppose you enlarge the picture in a copier so that it is 120% of its current size. How tall would each person, the desk, and the cabinet be in the enlarged picture?

MATERIALS

Ruler

Casio *Algebra FX 2.0* Graphing Calculator

EXTENSION

Draw a picture of a right foot 4 inches long. Cut it out. Measure the length of the foot of one of the members in your group. Now construct an entire model of this person.

ONE SOLUTION TO PROBLEM 2: GET THE PICTURE?

A. From left to right, the five people in the picture, which was taken by a Casio 8000 digital camera, are Jeff, Gwen, Jill, Joycelyn, and Ryan. (After the copying process, you can't always tell how good looking these people really are!) All are graduate assistants at Clemson University. Gwen is five feet, three inches tall. Use this information to determine the heights of the other four people. What assumption(s) must you make?

If we assume that all are standing up straight, that the copying process has not made any inconsistent distortions, and that we can measure accurately, (assumptions that may be violated), then we can use proportions to determine the heights of all five.

First, we must measure how tall they appear on the picture. Simply determining where to measure on a two-dimensional representation is difficult. Where is the point directly below the top of the head? Answers may vary, but the authors have obtained the following results:

?? Jeff: 87 mm

?? Gwen: 74 mm

?? Jill: 75 mm

?? Joycelyn: 70 mm

?? Ryan: 83 mm

Using Gwen's actual height, we can then set up four proportions to determine the others' heights. Since 12 inches is equal to one foot, Gwen is 63 inches tall.

$$\frac{74mm}{63inches} ? \frac{87mm}{Jeff} ? \frac{75mm}{Jill} ? \frac{70mm}{Jocelyn} ? \frac{83mm}{Ryan}$$

Solving, we obtain, $Jeff \ ? \ \frac{87mm \ ? \ 63inches}{74mm}$; $Jill \ ? \ \frac{75mm \ ? \ 63inches}{74mm}$;

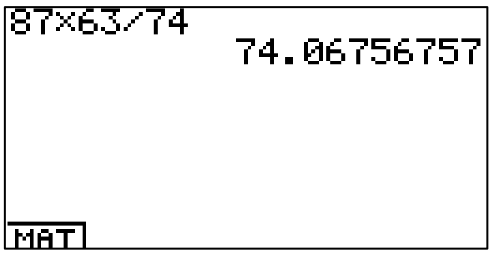
$Jocelyn \ ? \ \frac{70mm ? 63inches}{74mm}$; $Ryan \ ? \ \frac{83mm ? 63inches}{74mm}$.

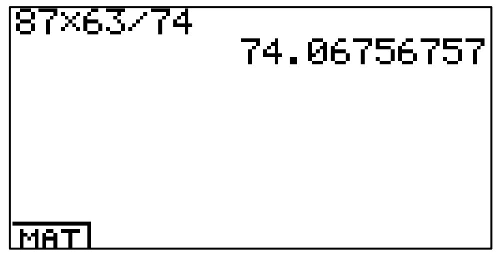
The calculations can be done simply in the RUN-MAT mode on the calculator. We will use the "replay" feature on the calculator to make the work even faster. If the screen needs to be cleared, press [AC/on] to remove any previous entries.

x Type in the first problem, and press EXE as shown below left. This tells us that Jeff is about 74 inches, or 6 feet 2 inches tall.

x Next press the right cursor key (on the disc), use the DEL key to delete the 87 and change it to 75, then press EXE as shown below right to find that Jill is about 63 inches, or about 5 feet 3 inches tall.

x Continue to press the right cursor key, change the first number as needed and press EXE .





After we continue with this procedure, we find the following:

?? Jeff is approximately 74 inches, or 6 feet 2 inches.

?? Jill is approximately 63 inches, or 5 feet 3 inches.

?? Jocelyn is approximately 60 inches, or 5 feet 0 inches.

?? Ryan is approximately 71 inches or 5 feet 11 inches.

It's very tempting to give you the heights that these four report for themselves, but we will leave the discussion as to the accuracy of these results (and your results) up to you.

B. A desk is 80 centimeters tall. How tall would it be in the picture? What about a filing cabinet that actually stands 134 centimeters?

We can use the same techniques we just used if we make sure that we are consistent in units. Before setting up the proportions, we'll change the height of the desk and cabinet into inches. We must first know that 2.54 centimeters is approximately equivalent to one inch. We can then simply divide 80 and 134 by 2.54. Alternatively, we can convert to inches by multiplying by a clever form of one. To convert the height of the desk into inches, we can do the following: $\frac{80cm}{1} ? \frac{1in}{2.54cm}$.

Note that the centimeters "cancel" so we're left with inches; our result is equivalent to 80 centimeters, however, because we have only multiplied 80 centimeters by 1. Once we have obtained our heights in inches we can proceed. The desk is about 31.5 inches high and the cabinet about 52.8 inches high.

Now we can set up our proportions, using the actual height as the numerator and picture height as the denominator: $\frac{63in}{74mm} ? \frac{31.5in}{desk} ? \frac{52.8in}{cabinet}$. Solving, we find $desk ? \frac{31.5in ? 74mm}{63in} ? 37mm$ and $cabinet ? \frac{52.8in ? 74mm}{63in} ? 62mm$.

C. Suppose you enlarge the picture in a copier so that it is 120% of its current size. How tall would each person, the desk, and the cabinet be in the enlarged picture?

We could set up proportions for all of these. Alternatively, we could simply multiply each of the picture heights by 1.2 to obtain the enlarged heights. Rounded to the nearest millimeter, results are as follows:

?? Jeff: 104 mm

?? Gwen: 89 mm

?? Jill: 90 mm

?? Jocelyn: 84 mm

?? Ryan: 100 mm

?? Desk: 44 mm

?? Cabinet: 74 mm

PROBLEM 3: PROPORTIONS IN THE WORLD

Solve the following problems in two different ways. First, set up and solve a proportion. Second, calculate the unit rate for the problem; then, set up a graph and a table for the problem and verify your earlier solution using both the table and graph.

A. In 1984-85, Michael Jordan scored 2,313 points in his rookie season to lead the National Basketball Association. If he scored this amount in 82 games, in which he played a total of 3,144 minutes, how many points did he average per game? On average, how long did it take him until he scored his 10^{th} point in a game?

B. Florence Joyner-Kersee, often called FloJo, set the world's record for women in the 100-meter dash in the trials for the 1988 Olympics with a time of 10.49 seconds. She went on to win three gold medals in the Olympics. If she could have kept that pace, how long would it have taken her to run a mile? How far could she have run in an hour?

C. In 1936, Jesse Owens became a hero by winning four Olympic gold medals. His time in the 100-meter dash was 10.3 seconds, and his time in the 200-meter dash was 20.7 seconds. Determine his average speed in the two races, expressing your results in miles per hour.

D. In baseball, Pete Rose had a career batting average of .303. (This means that, on average, he had 303 hits for every 1000 official times at bat.) How many hits would he expect to get in a season if he averaged 586 at bats in the season? Rose was major league baseball's all-time hit champion, ending his career with 4,256 hits. Using the information given, determine how many years he played in the majors.

E. On a map of South Carolina, 2.4 centimeters represents 25 miles. The distance between Clemson, SC, and Columbia, SC, measures 11 cm on the map. How far apart are the two towns? Would a car ride between the towns be this distance, shorter, or farther? Why?

F. If you drink six carbonated beverages per week, how many drinks would you consume over 10 years? How many ounces would you have had, assuming the drinks were 12 ounces?

G. Create at least three real-world situations involving proportions.

PROBLEM 4: CAPTURE-RECAPTURE

Marine biologists estimate the population of many species in the wild. For example, Samuel Gruber from the University of Miami in Florida has estimated the number of lemon sharks in the Bimini Lagoon. In this problem you will conduct a simulation for estimating such a population. Before you begin, brainstorm different methods a marine biologist might use for making such an estimate. Share your ideas with your classmates.

We'll now simulate one method of making a scientific estimate of such a population.

A. Take a shoebox filled with small cubes (oyster crackers, fish crackers, wrapped candies, and other such things can work too). Open the lid and estimate the number of cubes you believe are in the box. (This represents your guess as to the population of lemon sharks.)
B. One person in your group should now pull out a handful of cubes (capture a sample of the population) and mark them in some way. Depending on what you've filled the box with, you might use a marker or a sticker to identify those you have chosen.
C. Return the captured "critters" to the shoebox (to simulate returning the tagged animals to the wild). Shake the box to mix the contents thoroughly (simulating the mixing that would occur in nature).
D. Have each member of your group pick a handful from the shoebox (recapturing a sample from the population) and note the number of "critters" that have been tagged and the total number in the handful. Repeat this several times, being sure to shake the box before each person has picked a handful.
E. Use all of the information you have collected to estimate the size of the population. (Suggestion: the RUN-MAT and STAT menus of the *Algebra FX 2.0* can be very helpful.)
F. Finally, count all of the "critters." Compare this with your initial estimate and then with your most scientific estimate.
G. Comment on how effective you think this technique might be. Include in your discussion the cautions that must be taken so the animals are not put at risk.

REFERENCE: Understanding Rational Numbers and Proportions, NCTM Addenda Series, 1996, pp. 57-60.

PROPORTIONAL REASONING

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AW – Foundations of Algebra and Geometry (1998)	2.1; 4.1; 6.1-2
Glencoe – Mathematics Applications and Connections C1 (1995)	2.6; 3.9-10; 6.10-11; 10.1-2; 11.3; 13.5
Glencoe – Mathematics Applications and Connections C2 (1995)	3.7; 4.7; 6.4; 11.1-5; 12.6; 14.3-5
Glencoe – Mathematics Applications and Connections C3 (1995)	5.6; 6.6; 9.1-3; 9.5; 10.7; 11.3; 13.5-8
Houghton Mifflin – The Mathematics Experience I (1992)	5.7; 5.11; 7.1; 9.2-6; 9.8l 9.10-11; 11.1-2; 11.6; 11.8; 15.5
Houghton Mifflin – The Mathematics Experience II (1992)	1.10; 3.6; 3.8; 4.5; 9.3-4; 9.6-7; 10.6; 10.12; 10.14-15; 14.4
McDougal Littell – Gateways to Algebra and Geometry (1994)	3.2; 4.2-6; 5.1; 8.2; 11.7
Prentice Hall – Middle Grades Mathematics C1 (1995)	1.7; 5.4; 5.6-7; 7.10; 9.1-3; 9.5; 9.7; 9.11
Prentice Hall – Middle Grades Mathematics C2 (1995)	3.10; 4.10; 8.2; 9.1-4; 9.9; 9.12
Prentice Hall – Middle Grades Mathematics C3 (1995)	5.2; 6.2-3; 6.8; 7.3; 8.1-6; 10.3
SFAW – Middle School Math C1, V1 (1999)	2.11; 4.3; 5.7
SFAW – Middle School Math C1, V2 (1999)	10.1-8
SFAW – Middle School Math C2, V1 (1999)	6.1-8
SFAW – Middle School Math C2, V2 (1999)	7.5-10; 8.5-6; 11.5
SFAW – Middle School Math C3, V1 (1999)	5.1-7
SFAW – Middle School Math C3, V2 (1999)	10.1-2
SFAW: UCSMP – Transition Mathematics, Part 1 (1998)	2.7; 4.2-3; 4.7
SFAW: UCSMP – Transition Mathematics, Part 2 (1998)	9.5; 10.3; 11.5-9; 12.1