CLEMSON MIDDLE SCHOOL MATHEMATICS PROJECT
UNIT 6: MEASUREMENT

PROBLEM 1: DOG PENS

Your dog, Browser, needs a pen in which to run. You purchase 200 feet of fence to build the pen. Because there is a long fence at the back of the yard, you only need to fence in three sides. What dimensions should you use so that Browser has the greatest amount of room? What dimensions would you use if, instead of having the greatest amount of room, the fenced in area has the longest run?

MATERIALS

Graph paper

Casio *Algebra FX 2.0* Graphing Calculator

EXTENSION

If no sides are fenced and you are not restricted to rectangular shapes, what is the greatest area you can enclose with 200 feet of fence? Support your answer.

ONE SOLUTION TO PROBLEM 1: DOG PENS

Students should be given plenty of opportunity to explore this problem. To begin, allow them to try on their own with graph paper. After a few minutes, the teacher may wish to show them at least one approach for getting started. Depending on the number of units available on the graph paper, teachers may suggest that students let one unit of graph paper represent 10 feet. Then students should draw three sides of a rectangle with a perimeter of 10 units and calculate the area of the rectangle. They should find as many different size rectangles as they can for which the perimeter of three sides is 200 units.

After students have explored this and shared their results, the teacher should help students analyze the problem. One way to study the results is through a table. If we think of our rectangle as needing one length of fence and two widths of fence, then the table below shows the possibilities when the width is restricted to multiples of 10.

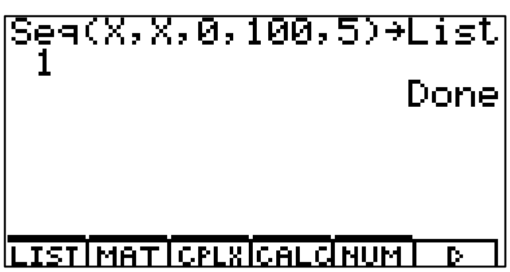
WIDTH (ft)	LENGTH (ft)	P (2W + L) (ft)	AREA (sq ft)
10	180	200	1800
20	160	200	3200
30	140	200	4200
40	120	200	4800
50	100	200	5000
60	80	200	4800
70	60	200	4200
80	40	200	3200
90	20	200	1800
100	0	200	0
110	***-20***	200	***-2200***

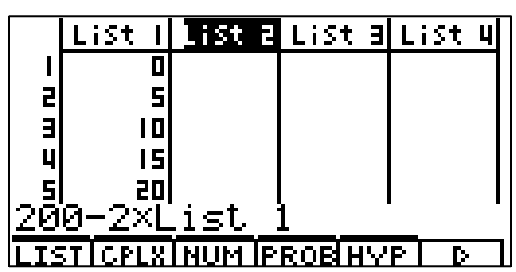
Although not all possibilities have been listed, from the table it appears that the maximum area that can be set up for Browser is 5000 square feet. This occurs when the two widths are set at 50 feet, resulting in a length of 100 feet.

We will now analyze the problem in more detail. We will create lists of possible widths and lengths in the calculators and then study the areas generated by them.

First, we will establish possible widths in List 1, but we'll be a little more complete than we were in the previous table. We could type in the set {0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100} into List 1, but we can do this more quickly if we prefer. (We know the width cannot be larger than 100 since there are two widths and only 200 feet of fence to use.) From the MAIN MENU, choose RUN-MAT. Then

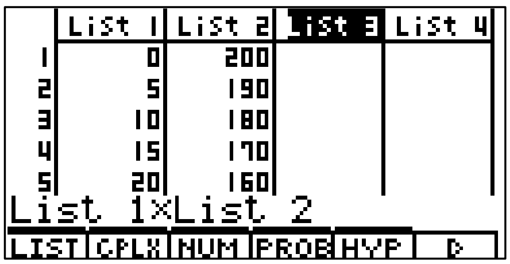
- Press OPTN F1 for LIST, and 3 for Sequence.
- Type in X (using the X,?,T key), a comma, X (using the same key as before), a comma, 0 (for the starting value), a comma, 100 (for the ending value), 5 (for the amount to skip by), right parenthesis, the store key (the right arrow key above AC/ON), F1 then 1 for List, then type 1 for List 1, and EXE . See below left.
- Press MENU to return to the MAIN MENU and call up STAT. You should see the multiples of 5 in List 1. Delete the numbers in List 2 (and List 3 for later work) if needed. We now wish to create the lengths in List 2. Use the right and up arrow cursor keys on the disc so that List 2 is highlighted.
- We want to subtract twice the numbers in List 1 from 200. Type in 200, the subtraction key, 2, the multiplication key, OPTN , F1 then 1 for LIST, then type 1 for List1. See below right. After you press EXE the lengths should appear in List 2.

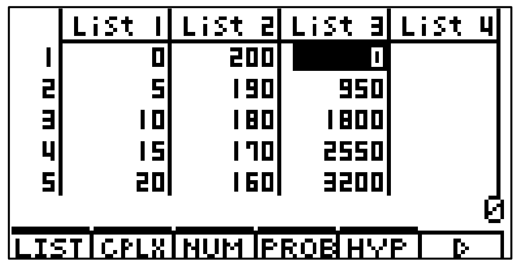




We now wish to show all of the areas in List 3. To do so,

- Move the cursor into List 3. Use the right and up arrow keys on the disc so that List 3 is highlighted. If List is no longer one of the selections available, press OPTN , F1 . Then press 1 for List, and type 1 for List 1, the multiplication sign, OPTN , F1 then 1 for List, and 2 for List 2. See below left. Press EXE to see the results. See below right.



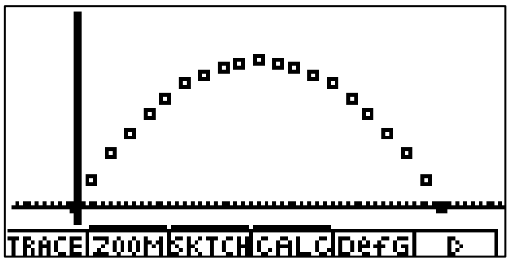


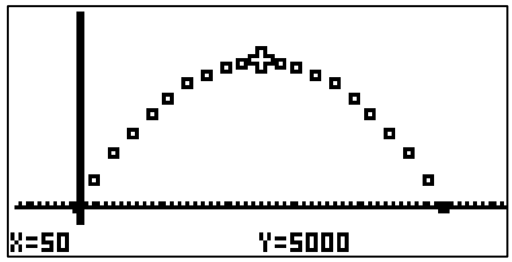
We are trying to determine the dimensions that will produce the maximum area. Let's look first at a scatterplot of the data.

- Press ESC to return to the primary statistics screen. It may be necessary to press F6 until GRPH is above F1 , then press F1 for graphs, and 5 for Set.
- Make sure StatGraph1 is at the top of the screen and if not, press F1 . Use the down arrow to highlight GraphType. Press F1 to select Scat.
- Use the down arrow to highlight Xlist and make sure List 1 is selected (pressing F1 , 1 then EXE if needed). Press the down arrow and press F1 , 3 then EXE so that the Ylist uses List 3. Then use the down arrow again to highlight Freq, and press F1 so that each point is used one time. Press ESC to return to the primary statistics screen.
- Make sure the window is set to automatic by pressing CTRL F3 . If it isn't, press F1 for Auto when StatWind is highlighted. Press ESC to return to the primary statistics screen and F1 then 1 to draw the scatterplot. See below left.

The horizontal component of the each point represents the width of the pen, and the vertical component represents the area. Consequently, to find the biggest area, we want to find the highest point.

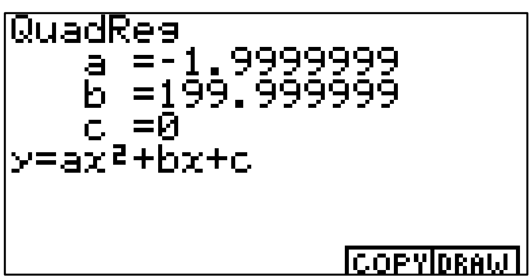
- Press F1 to be able to trace the scatterplot, and then use the right (and left) arrows on the disc to move from point to point. The highest point is (50, 5000), which again tells us to maximize the area of the pen, we should set the width at 50 feet. See below right.

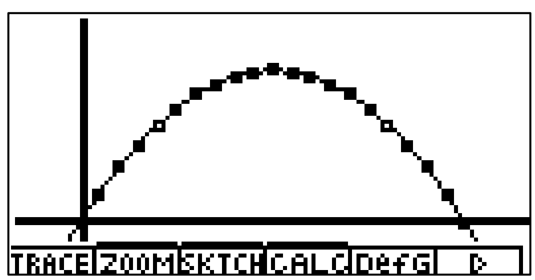




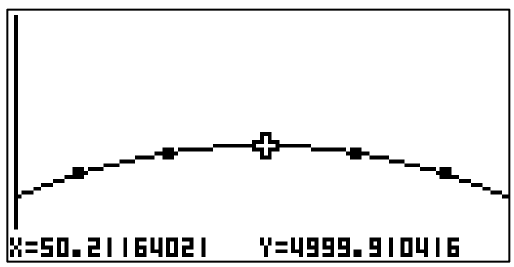
For students who may be more mathematically sophisticated, quadratic regression can also be done.

- If looking at the traced scatterplot, press ESC to end the trace, then press F4 for CALC, and 4 for Quad (quadratic regression). The result, shown below left, indicates that the relationship between the width and the area can be expressed by the function $y = -2x^2 + 200x$. (The values for *a* and *b* on the screen, -1.9999999 and 199.9999999, have been rounded.) Then press F6 to DRAW the function. See below right. Note the perfect fit.

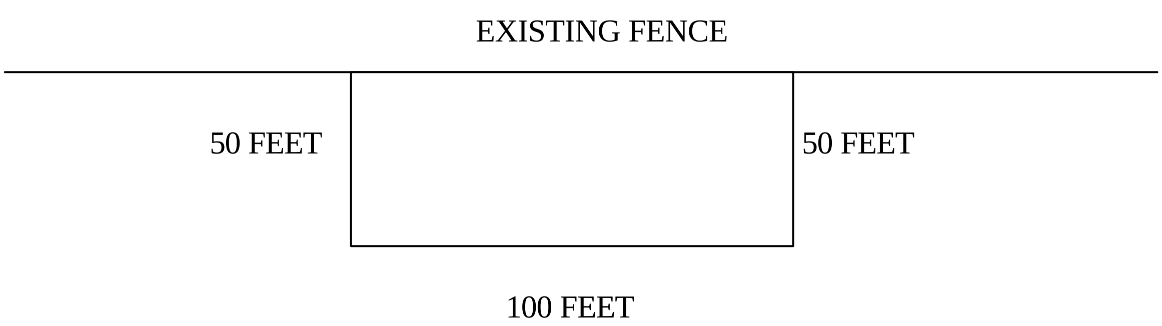




- Finding the highest point on the graph of this equation allows us to find the best solution to the problem, one in which the width does not have to be a multiple of 5 feet. First we will zoom in to the desired point, at the top of the graph. From the graph screen, press [F2] for ZOOM, then [3] for In, and use the disc of arrows to move the "+" cursor as close as possible to the highest point. Then press [EXE]. Next press [F1] for TRACE and notice that the highest point (when rounded) still appears to be (50,5000). See below.



Another option available is to study the function in a table that we can create on the calculator. This numeric solution should confirm the results we have found from our other techniques. We have found that, if we are to build a rectangular pen for Browser using the existing fence for one side of the pen, the most area we can give Browser is 5000 square feet. We will accomplish this by making two sides of 50 feet (our width) and one side of 100 feet. A rough sketch of this is shown below.



Finally, if instead of having the greatest amount of room (area), you wanted to determine the longest run, you would need to discuss the least possible width, w, (dependent on the size of the dog) for which the dog would run comfortably. The length would then be $200-2w$. Answers will vary as students will pick different values for w.

PROBLEM 2: BUILDING A BETTER BOX

Take a sheet of paper. Students should work in small groups trying to build a box without a lid that has the greatest possible volume. After this exploration, the teacher should begin to lead the students to an analysis of the problem.

Students should cut off four equal squares from the corners and then fold the paper into a box without a top. Groups should make several boxes. The goal is to determine the length of a side of the removed square that produces the box with the greatest volume. The problem should be analyzed algebraically, graphically, and numerically.

MATERIALS

Paper (graph paper may be helpful)

Tape

Rice Krispies or something similar that can be used to fill the created boxes

A graduated cylinder to compare the amounts the boxes hold

Casio *Algebra FX 2.0* Calculator

EXTENSION

Again try to create a box with the greatest volume, beginning with a sheet of graph paper. This time, however, use the single sheet of paper to create a box that has a top. Support your choice.

ONE SOLUTION TO PROBLEM 2: BUILDING A BETTER BOX

This solution is appropriate only after students have been given ample opportunity to explore the problem on their own. A contest among the groups in the classroom can be very powerful in helping students develop many concepts.

The solution provided here uses inches, but if centimeter graph paper is used, the metric system may make the analysis clearer. If we use a normal piece of paper for the box, the dimensions are $8\frac{1}{2}$ inches by 11 inches. From this we are to remove four congruent squares from the corners. We will then fold the edges up into a box without a lid.

Suppose we cut squares one inch on a side from the four corners. Both the width and length will be reduced by 2 inches and the box will have a height of one inch. The volume can be found by multiplying our new dimensions, $6\frac{1}{2} ? 9 ? 1$, or 58.5 cubic inches. This, of course, is just one possible box. Before we use the calculator to speed things up, let’s look at the beginnings of a table of information. Note that we couldn’t cut out a square five inches on a side; the paper isn’t big enough.

SIDE OF SQUARE (inches)	**WIDTH (inches)**	**LENGTH (inches)**	**VOLUME (cu in)**
1	6.5	9	58.5
2	4.5	7	63
3	2.5	5	37.5
4	0.5	3	6

Of the boxes shown here, the one with two-inch squares cut out is clearly the best. However, we have only looked at whole number possibilities.

Once students understand what the table represents, they may be ready to start taking advantage of some of the calculator’s capabilities. This will not only speed things up but allow us to investigate more of the possibilities.

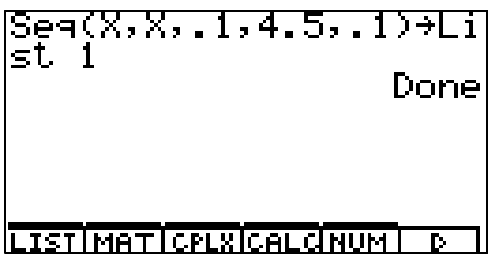
We'll use a sequence to create a list of possible side lengths for the squares in List1. Once we have this, we will put the resulting widths in List 2, the resulting lengths in List 3, and the volumes in List 4.

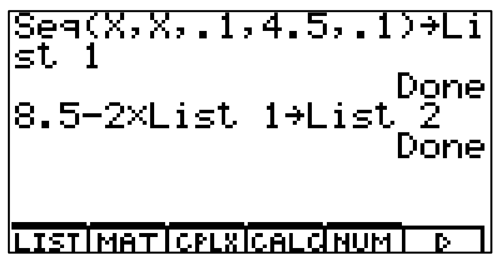
From the MAIN MENU, choose RUN-MAT. To create a set of possible squares to remove in List 1,

- Press [OPTN], [F1] for LIST, then [3] for sequence.
- Type in X (using the [X,?,T] ey), a comma, another X, a comma, 0.1 (for the starting value), a comma, 4.5 (for the ending value), 0.1 for the stepping value, the right parenthesis, the store key (the arrow above [AC/ON]), [F1] for then [1] for List, and [1] for List 1. Then press [EXE]. See below left.

Rather than moving to the STATISTICS menu, we will continue to create our lists from the RUN-MAT menu.

- To store the widths of the box in List 2, type in 8.5, the subtraction sign, 2, the multiplication symbol, [F1] then [1] for List, [1] for List 1, the store key (the arrow above [AC/ON], [F1] then [1] for List, and [2] for List 2. Then press [EXE]. See below right.





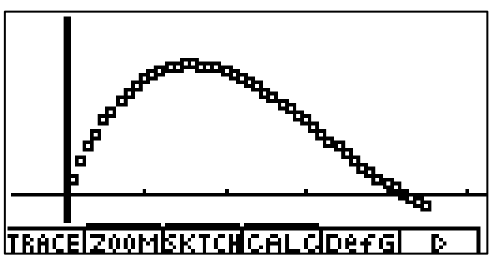
Use a similar strategy to store the lengths (11 minus twice List 1) in List 3. Finally, store List 1 times List 2 times List 3 in List 4 for the volumes of the boxes.

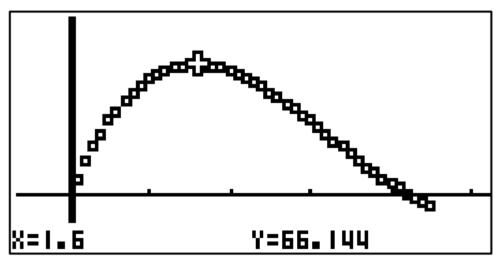
If we go to the STAT menu from the MAIN MENU, we can now scan through our data, looking for the largest value in List 4. Keep in mind, however, that we increased the sides of the square by one-tenth of an inch each time. If we needed, we could increase the box lengths by smaller amounts to obtain even more precise results.

The largest value in List 4 is 66.144. This value, which represents 66.144 cubic inches, occurs when the side of each square that is removed is 1.6 inches. To see this, move to the left from 66.144 to read the corresponding value in List 1.

We now wish to look at a graph of our scatterplot. If you are not already there, from the MAIN MENU, choose STAT.

- Choose F1 for graph and 5 for SET.
- Make StatGraph1 a scatterplot with List 1 for the Xlist, List 4 for the Ylist , and 1 for the frequency. Press ESC when all selections have been made.
- Press F1 then 1 to draw the scatterplot. See below left. (If necessary, go to CTRL F3 first to set the StatWind to Auto.)
- Press F1 to trace the scatterplot, seeking the highest point. You should gain confirmation that the greatest volume of 66.144 cubic inches is obtained when squares of sides 1.6 inches are removed.





Keep in mind that we have only investigated squares with sides rounded to the nearest tenth of an inch. Similar techniques can be used, however, to obtain more precise results. For example, when setting up the values for List 1, you might select sides to the nearest hundredth of an inch with the command Seq(X,X,1.5,1.7,0.01). This would generate numbers between 1.5 and 1.7, inclusive, that count by hundredths.

We have used algebra in creating our lists, but an even more algebraic approach can be taken to this problem.

Let x represent the length of the side of the squares that are removed. Then

?? x also represents the height of the box

?? $8.5 ? 2x$ represents the width of the box

?? $11 ? 2x$ represents the length of the box

?? $x(8.5 ? 2x)(11 ? 2x)$ represents the volume of the box

By graphing and exploring this function, we could once again determine the length of the sides of the squares that should be removed to form a box with the greatest volume.

PROBLEM 3: HOW WELL CAN YOU ESTIMATE?

Consider items 1 – 5, which your teacher will identify for you, and figures 6-10, which are drawn on the following pages. For each item, estimate the given attributes, including units. Then, using a ruler, a balance scale, a protractor, and centimeter grid paper, measure the attributes. Finally, answer the questions below.

ITEM	**MEASURED LENGTH**	**ESTIMATED LENGTH**	**MEASURED WEIGHT**	**ESTIMATED WEIGHT**
PEN				
KEY				
BOOK				
CALCULATOR				
QUARTER				

FIGURE	**MEASURED ANGLE A**	**ESTIMATED ANGLE A**	**MEASURED AREA**	**ESTIMATED AREA**
6				
7				
8				
9				
10				

A. Construct four scatterplots, one for each attribute, that compare the measured attribute with your estimate. Be sure to use and show an appropriate scale.

B. On each graph, draw the line $y ? x$.

C. For each attribute, determine if you over-estimate, under-estimate, or are not consistent. How can you tell from the graph?

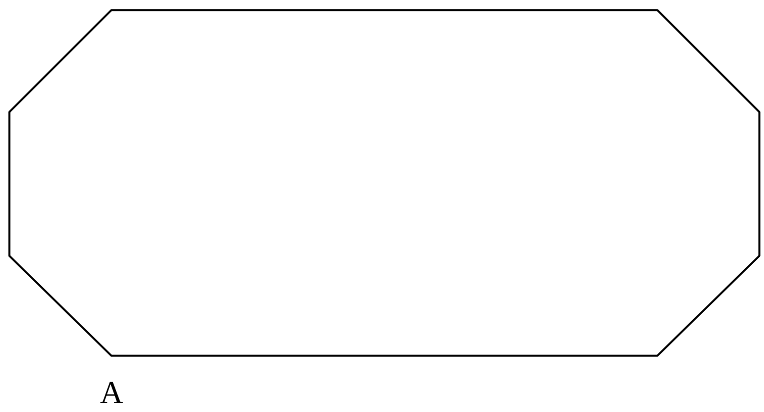
D. Why do you think the "measured" column was listed first, even though you made your estimate before you measured?

E. List the attributes in order of your ability to estimate them, beginning with the attribute you estimated best. How did (could) you determine this from the graphs?

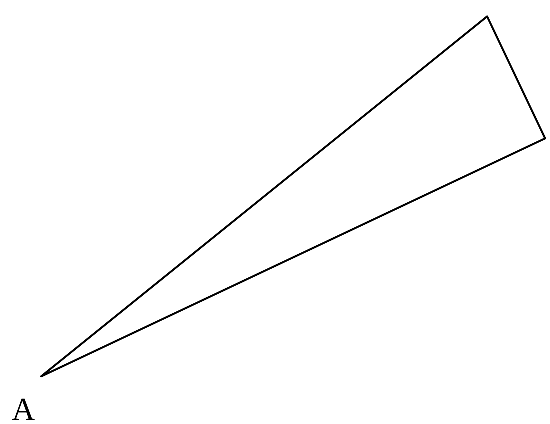
F. Calculate the deviations between your estimates and the measures you obtained. What do positives and negatives indicate here? How could you determine the deviations from the graphs?

G. Calculate the percent of error for your estimates. Does this affect your opinion as to the order of attributes you listed in part E? Explain.

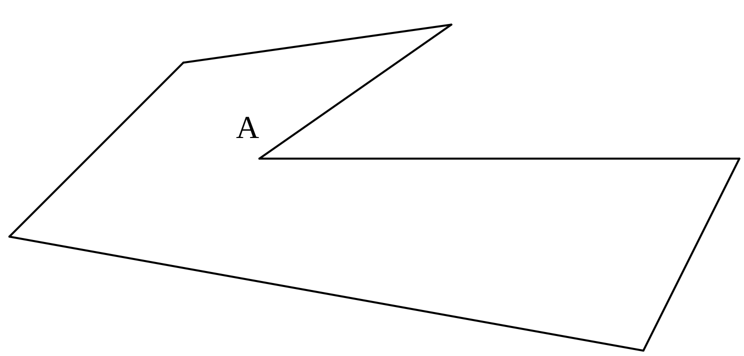
ITEM 6



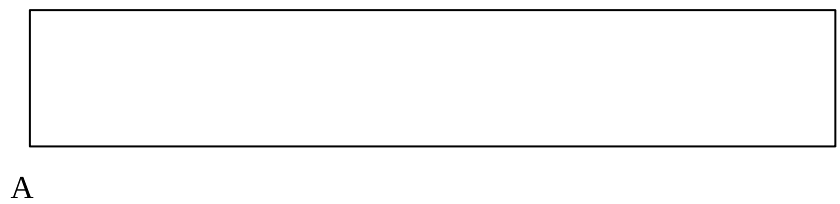
ITEM 7



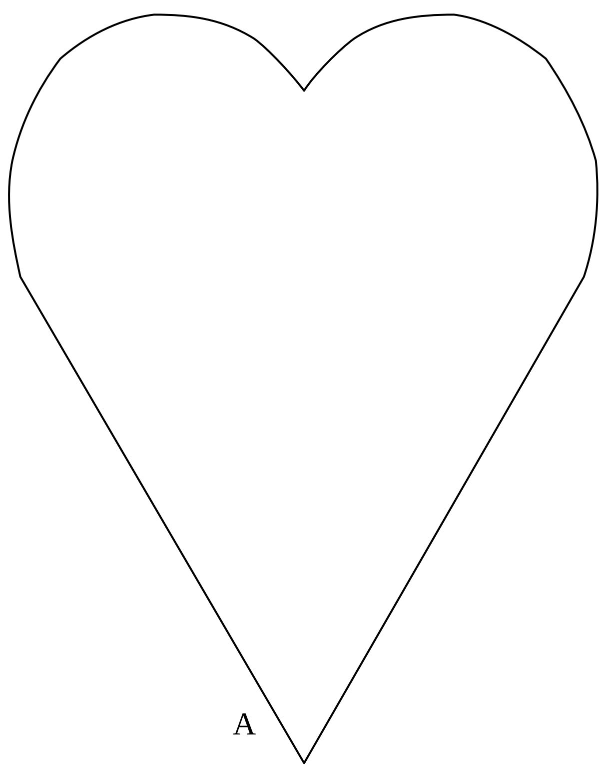
ITEM 8



ITEM 9



ITEM 10



PROBLEM 4: TEMPERATURE CHANGES

What do you think happens to the temperatures of cold and warm liquids as they sit out in a room? Think about this question and discuss it with a partner before beginning this exploration. After you have arrived at a conjecture, you are ready to proceed.

First take the temperature of the room and record the results. Pour a cold liquid, but one without ice, into a paper cup, and set it out in the room. Take its temperature once every minute for 50 minutes. Construct a scatterplot, using the horizontal axis for time and the vertical axis for temperature. Draw a horizontal line on your scatterplot that shows the room temperature. Write a paragraph about what you have found. Repeat the problem for a hot liquid. Compare and contrast your results.

MATERIALS

Cold liquid in cup
EA-100 and temperature probe
Casio *Algebra FX 2.0* Graphing Calculator

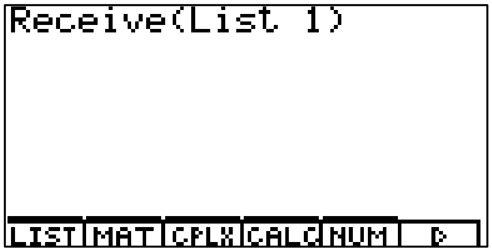
GETTING STARTED

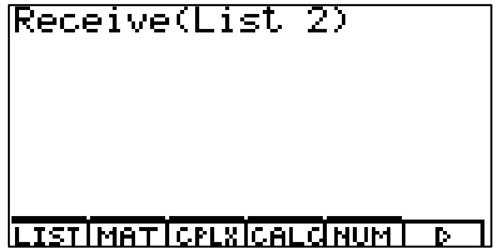
The EA-100 and the temperature probe that comes with it can be extremely helpful for this problem. The instructions to gather the data and then transfer the data to your calculator follow. Be sure both the EA-100 and FX 2.0 are turned off before you begin.

1) Gathering the data
   - Plug the temperature probe into Channel 1 of the EA-100.
   - Turn the EA-100 on.
   - Press [SHIFT] [MODE] to enter the SET UP for the data logger.
   - The first setting indicates how often the data should be sampled. Press [DataLOG] repeatedly until 60.0 seconds is shown. This tells the EA-100 to sample data once every minute. Press [TRIGGER] to set this.

- The next entry tells the EA-100 how many times to sample the temperature. Press [DataLOG] repeatedly until 50 is shown. This tells the EA-100 to sample data 50 times. Press [TRIGGER] to set this.
- The next setting tells the EA-100 how to record time. The setting we want is 1, so that time is recorded as "real time." If necessary, press [DataLOG] until 1 is displayed. Press [TRIGGER] to set the time.
- At this point, the EA-100 should display READY on the left side of the screen. If not, work back through the previous steps to make sure you have set things up correctly.
- Put the temperature probe into the cold liquid.
- Press [TRIGGER] to begin sampling data. During data collection, the message "SAMPLING" will flash on and off on the EA-100. When all of the data have been collected, the EA-100 will display "Done."

2) To transfer the data from the EA-100 to the calculator
   - Connect the EA-100 to the *FX 2.0* with the black cable.
   - On the calculator, select RUN-MAT from the MAIN MENU.
   - Press [SHIFT] [VARS] (for program).
   - Press [F5] for I/O and [4] for Receive.
   - Press [OPTN] , [F1] then [1] for LIST, then [1] again for List 1, and the right parenthesis. See below left. Press [EXE] . This will transfer the time values into List 1.
   - Press [AC/ON] and the up arrow on the disc to reshow the previous command. Use the left arrow until the 1 in List 1 is highlighted. Type in a 2 and delete the 1, so that the calculator shows Receive(List 2). See below right. Press [EXE] . This will transfer the temperatures into List 2.





With the data in your calculator, you should be ready to explore the problem. The times will be in List 1, the temperatures of the liquid will be in List 2. If you prefer, you can put the data into different lists. Just keep in mind that the first receive command transfers the times and the second receive command transfers the temperatures. .

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AW – Foundations of Algebra and Geometry (1998)	1.2-3; 2.3; 3.3; 4.1; 5.2-3; 6.2; 8.1-3
Glencoe – Mathematics Applications and Connections C1 (1995)	2.6; 3.3; 3.10; 4.6; 4.7; 11.6; 13.5
Glencoe – Mathematics Applications and Connections C2 (1995)	2.9; 3.7; 5.7; 6.4; 6.7; 10.5; 14.3-4
Houghton Mifflin – The Mathematics Experience I (1992)	8.2; 8.13; 14.2
Houghton Mifflin – The Mathematics Experience II (1992)	7.5; 14.3
McDougal Littell – Gateways to Algebra and Geometry (1994)	3.1; 4.1; 5.3; 11.7
Prentice Hall – Middle Grades Mathematics C1 (1995)	1.7; 5.4; 5.6-7; 6.2
Prentice Hall – Middle Grades Mathematics C2 (1995)	1.5; 3.10; 5.2; 5.9; 6.7
Prentice Hall – Middle Grades Mathematics C3 (1995)	1.6; 4.7; 5.2-3; 6.2-3; 6.6; 11.5-6
SFAW – Middle School Math C1, V1 (1999)	1.3; 2.11; 4.1; 4.4; 4.7
SFAW – Middle School Math C1, V2 (1999)	8.3; 9.7; 11.7
SFAW – Middle School Math C2, V1 (1999)	1.5-6; 2.1; 2.3; 2.5; 5.1; 5.5
SFAW – Middle School Math C2, V2 (1999)	10.2-5; 11.4
SFAW – Middle School Math C3, V1 (1999)	1.7; 3.1-3; 4.1; 4.3
SFAW – Middle School Math C3, V2 (1999)	8.4; 9.1; 9.6; 10.1; 10.3; 10.5
SFAW: UCSMP – Transition Mathematics, Part 1 (1998)	3.6; 3.8-9; 4.2-3; 4.7
SFAW: UCSMP – Transition Mathematics, Part 2 (1998)	8.4; 9.2; 12.4; 12.7; 13.1