CLEMSON MIDDLE SCHOOL MATHEMATICS PROJECT
UNIT 8: SEQUENCES

PROBLEM 1: LINEAR MOTION

For this investigation, you will need your graphing calculator, an EA-100 data collector, and a motion detector. Make sure the MOTLIN (Motion – Linear) program, along with its subsidiary programs (MOTLIN1-5), has been loaded on your calculator. Then, call up the program and explore "PLOT A WALK." Once you feel comfortable with that, select "WALK BY GRAPH." Answer the questions posed to you about a graph and then try to "walk" the graph. Work through an analysis of the graph and develop an equation for it. Complete this activity several times. Discuss what you have discovered about the relationship between the graph, what you need to do to match it with your walk, and its equation.

MATERIALS

Casio *Algebra FX 2.0* Graphing Calculator

EA-100 Data Collector

Motion Probe

ONE SOLUTION TO PROBLEM 1: LINEAR MOTION

The MOTLIN program has been designed to help students understand important ideas related to linear functions. The program uses the x-axis as time (measured in seconds) and the y-axis as distance from a motion detector (default measures are in meters).

First, make sure that each calculator that will be used with the data collector has the series of six programs entitled MOTLIN, MOTLIN1, MOTLIN2, MOTLIN3, MOTLIN4, and MOTLIN5. To transfer these programs from one calculator to another, make sure the calculators are connected with the supplied cable. Once connected, turn the calculators on, and select LINK from the MAIN MENU on both the receiving and sending calculators. Then,

- Press [F2] on the receiving calculator.
- Press [F1] on the sending (transmitting) calculator.
- Press [1] to select the desired programs from the sending calculator.
- Use the down arrow and [EXE] to select each of the six programs named above.
- Press [F6] to transmit. If you get an error message, make sure the cable is securely attached to both calculators.

Before the students begin to work on the problem, the teacher should introduce the program with a demonstration for the entire class. First, all of the tools must be hooked up correctly, with the EA-100 turned off.

- Set the switch on the top of the motion detector on wide range. Depending on the physical arrangement of the room, this may have to be reset on narrow range. This will be necessary if the motion detector picks up the wrong things along the borders of the path.
- Clear a path about 2 meters wide and 8 meters long. This will be used for a person to walk toward and away from the motion detector.
- Using the cable, attach the calculator to the EA-100 data collector.
- Turn the EA-100 on.

- Connect the EA-100 to the motion detector with the supplied cable. The cable plugs into the sonic port on the EA-100. A green light on the detector should begin flashing. Set the motion detector along the walking path at a height that will "hit" the walker.
- Turn the calculator on. From the MAIN MENU, call up PROGRAM.
- Highlight MOTLIN and press EXE .

The menu screen for the program is shown below. MOTLIN has been set to collect 20 data points (N=20), collecting one every half second (DT=0.5). Distance from the detector is measured in meters (U=[M]). These default values can be changed by typing in 8 to select OPTIONS from the menu, pressing EXE , and making the desired changes, but it is recommended to begin with the default values.

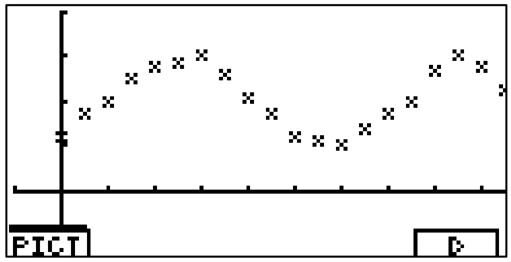
First, we'll explore "PLOT A WALK."

- From the menu screen shown below, press 1 and EXE .
- Follow the directions to plot a walk. These tell you to press the TRIGGER key on the EA-100 when you are ready to start and the EXE on the calculator when the sampling is finished.

```
?
 1 PLOT A WALK
 2 WALK BY GRAPH

 8 OPTIONS    9 EXIT
N=20  DT=0.5   U=[M]
```

After you have finished, a graph of the walk should be displayed on your calculator. Below is one particular walk. The teacher should point out that the tick marks on the *x*-axis represent seconds and the tick marks on the *y*-axis represent meters away from the detector. The teacher should then lead a class discussion helping students to interpret the graph.



Some issues and questions that the teacher may wish the class to discuss follow.

?? Let’s make sure everyone understands what a point represents. For example, what does the second point to the right of the *y*-axis represent? (One second after the trigger was pressed, which started the data collection, the walker was about 2 meters away from the detector.)

?? Which direction did the walker move when the program started? (Away from the detector, since y-values are getting larger.)

?? How far from the detector did the walker start? (A bit more than 1 meter, because the graph appears to have a *y*-intercept just above 1.)

?? What direction is the walker moving when the graph goes up? (Away from the detector – the distance is becoming greater.)

?? What direction is the walker moving when the graph goes down? (Toward the detector – the distance is becoming smaller.)

?? When did the walker change directions? (Approximately 3, 6, and 8.5 seconds into the data collection – this is based on when the graph changes from increasing to decreasing, that is from up-hill to down-hill, and vice-versa.)

?? What would a straight, but slanted line, represent? (Motion at a constant speed.)

?? What would a curved graph represent? (Motion at changing speeds.)

?? What would a horizontal line represent? (Standing in the same place, no motion.)

?? Could you “walk” a circle with this program? (No, you can’t be in two places at the same time.)

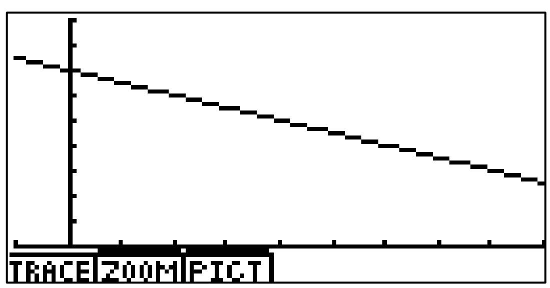
?? Are there shapes that cannot be “walked”? What are they? What do these shapes have in common? (Answers may vary. A key idea is that for any input there is only one output – one definition for function. Closed curves are therefore impossible to “walk” on the screen, or any type curve in which a vertical line could pass through in more than one place at a time.)

If for some reason a mistake is made while collecting data and the AC/ON key is pressed to break out of the program, you will need to unplug the EA-100 and reset the equipment as described earlier.

Once they have seen a demonstration of "PLOT A WALK," students should be given ample opportunity to explore it on their own. After you consider the physical size of the room, the number of students, and, of course, the number of equipment set-ups available, divide the class into groups. Groups of three or four often work best, but groups of two can work too. Each person should be assigned roles. For example, for a group with four people, one person can be the "techie," assigned to make sure the equipment is hooked up properly and the path for the walker is clear. One person can be the "trigger person," giving the commands to the data collector and the calculator. A third can be the "commander," telling the walker where, when, and how fast to go. The fourth person can be the "walker." Group members should rotate through these roles..

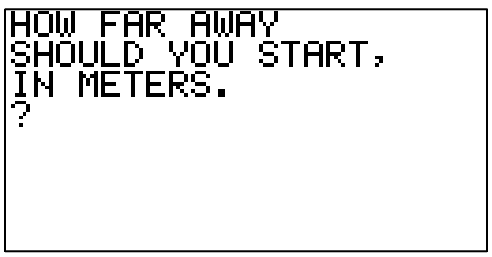
When students are comfortable with "PLOT A WALK," they are ready to move on to the second component of the MOTLIN program, "WALK BY GRAPH."

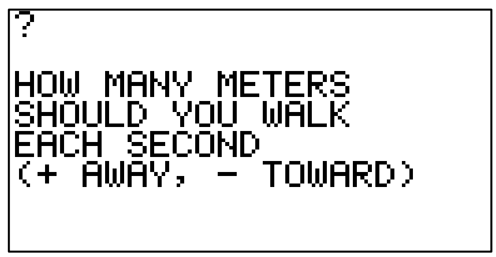
- To begin, from the MOTLIN menu press 2 and EXE . The calculator will prompt you to study a graph. Press EXE to display the graph that has been chosen randomly from a small selection. One such graph is shown below.



The group should discuss how to match this graph by walking toward or away from the motion detector. For the graph shown above, the person should start about 7 meters away from the motion detector and then move towards the detector at a constant speed, reaching about 3 meters away from the detector in 8 seconds.

- Once the group has decided what the walk entails, press EXE on the calculator. The calculator will ask a couple of questions about the graph. Read each question, and type in the answer you believe appropriate, and press EXE . The first question, shown below left, asks how many meters from the detector you should start the walk. Put in the number you think and press EXE . The calculator will give you feedback. For the graph shown above, the answer is 8.
- The next question asks how many meters you should walk each second (speed). See below right. Based on your understanding of the graph, type in your answer and press EXE . Again, the calculator will give you feedback. For the graph shown, the speed should be -0.5 meters each second. The negative is used because as you move toward the detector, the distance from it decreases.





- After the calculator has provided feedback on the two questions, pressing EXE will redisplay the graph. Press EXE again when you are ready to proceed.
- The calculator will prompt you to press TRIGGER on the EA-100 when you're ready to walk and EXE on the calculator when the data collection is finished. Before starting, note that the display on the data collector shows how far away the person is, so the starting point can be determined fairly accurately.
- After you have completed your walk, the calculator will display the original graph and a plot of your just-completed walk. Press EXE and the calculator will ask you if you want to try again. Type in 1 or 0, as desired, then press EXE .

x Once you decide not to try again, the program can help you analyze the graph. You can skip this part if you wish, but there is a lot of rich mathematics here.

Press 1 and EXE to begin the analysis.

Through the analysis the calculator tells you where you should be at each time. For example, for the graph pictured earlier, the calculator tells you that,

?? At 0 seconds, you should be 7 meters away.

?? At 1 second, you should be 6.5 meters away.

?? At 2 seconds, you should be 6 meters away.

?? At 3 seconds, you should be 5.5 meters away.

?? At 4 seconds, you should be 5 meters away.

?? At 5 seconds, you should be 4.5 meters away.

?? At 6 seconds, you should be 4 meters away.

?? At 7 seconds, you should be 3.5 meters away.

?? At 8 seconds, you should be 3 meters away.

?? At 9 seconds, you should be 2.5 meters away.

Let's look at this sequence of distances and see if we can determine an equation that relates your distance from the detector with the amount of time you have walked.

SECONDS	**DISTANCE**
0	7
1	7 ? .5 ? 1 ? 6.5
2	7 ? .5 ? 2 ? 6
3	7 ? .5 ? 3 ? 5.5
4	7 ? .5 ? 4 ? 5
5	7 ? .5 ? 5 ? 4.5
6	7 ? .5 ? 6 ? 4
7	7 ? .5 ? 7 ? 3.5
8	7 ? .5 ? 8 ? 3
9	7 ? .5 ? 9 ? 2.5
x	7 ? .5 ? x ? y

The bottom row in the table is actually a formula that relates distance walked, *y*, in meters, to the time, *x*, in seconds. You should start 7 meters away; at any point in time after this, the distance you should be from the detector is 7 minus the amount you've walked. That amount walked is 0.5 (the number of meters you should walk each second) times the number of seconds you have been walking. Again,

?? Distance from the detector = Starting distance – distance walked (the minus assumes we're walking toward the motion detector, reducing the distance).

?? Stated mathematically, we have *y* ? 7 ? .5*x*

We can use this to build a more general form for our equation. First, let's define the meaning for several letters.

?? Let *a* represent the distance that we should start at (mathematically this is called the y-intercept)

?? Let *b* represent the rate at which we should walk. Subtraction can represent movement toward the detector (since the distance is getting smaller) and addition can represent movement away from the detector (because the distance is increasing).

?? Let *x* represent the number of seconds we've been walking.

?? Let *y* represent our distance from the motion detector, at any given time.

?? Then, a general form for our equation becomes *y* ? *a* ? *bx* or *y* ? *a* ? *bx*.

After the calculator helps you work through the sequence of distances, it displays the equation. Press [EXE] and the original graph will be displayed once more. Again press [EXE] to return to the beginning of the program.

By using this program, students should discover the key components that they will study in their high school work with linear equations. They should see how the sequence of distances from the detector can help them develop a formula that relates their distance to the time they have been walking.

PROBLEM 2: MEDICINE IN THE BLOODSTREAM

Suppose that due to an illness, you have to take 200 milligrams of a prescription every day for 21 days. Further suppose that your body disposes of 80% of whatever medicine is in your bloodstream every day. What do you think will happen to the amount of medication in your bloodstream over the 21 days?

A. Explore the amount of medicine in your bloodstream over the 21-day period. Comment on your findings.
B. If you forget to take your medicine one day, you could either skip the day or take a double dose. Explore the different amounts in your bloodstream for each of these two options. Then determine which you think is better. Why?
C. How long does it take the medicine to be completely gone from your bloodstream? Support your answer.

MATERIALS

Casio *Algebra FX 2.0* Graphing Calculator

EXTENSION

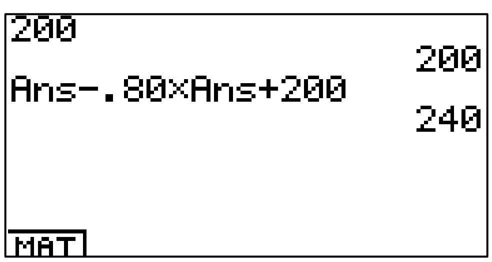
Change the amount of medication you take each day. Then change the percent of the medicine your body dissolves each day. Try to arrive at a method for determining the amount of medicine that will be in your bloodstream in the long run.

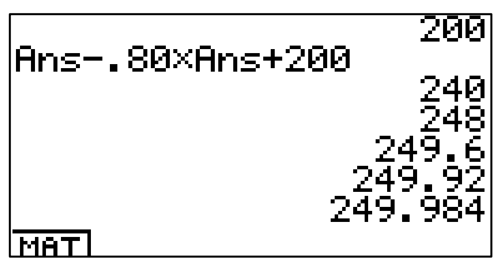
ONE SOLUTION TO PROBLEM 2: MEDICINE IN THE BLOODSTREAM

A. Explore the amount of medicine in your bloodstream in your bloodstream over the 21-day period. Comment on your findings.

Before beginning, students should think for a few minutes about the amount of medicine that would be in their bloodstream over the course of the three weeks. They may think that the amount of medicine in your bloodstream will keep increasing for as long as you take it. They are right, but not perhaps in the way they think. Only after they have thought about this and discussed it should they begin a more formal investigation. To begin, choose RUN-MAT from the MAIN MENU.

- Type in 200 to represent the initial amount and press [EXE] . This “seeds” the calculator.
- Over the course of the day, your body will deplete 80% of what is there, but then you take another pill. Press the subtraction key (the calculator automatically assumes you want to subtract this from the previous answer), .80, the multiplication key, [SHIFT] [(-)] for the Ans key, the plus key, 200, and [EXE] . See below left. This indicates that after you take your second pill, you have 240 mg of medicine in your blood stream. See below left.
- Continue to press [EXE] to represent each of the days. The first five days are shown below right.





You can continue to press [EXE] to represent each additional day. We will now organize our findings into a table. In the first column we will represent the number of days you have been taking the medication and in the second column the amount of medication in your bloodstream.

# OF DAYS	AMT (mg)	# OF DAYS	AMT (mg)
1	200	12	249.999999
2	240	13	249.9999998
3	248	14	250
4	249.6	15	250
5	249.92	16	250
6	249.984	17	250
7	249.9968	18	250
8	249.99936	19	250
9	249.999872	20	250
10	249.9999744	21	250
11	249.9999949		

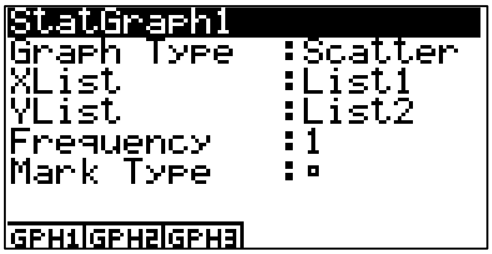
The amount of medication in your bloodstream does indeed increase, but at slower and slower rates. By the 14^{th} day, the calculator rounds this to 250 milligrams. Note that if we round to the nearest milligram, not an unreasonable thing to do, by the fourth day the amount of medication in your bloodstream has already stabilized at 250 milligrams.

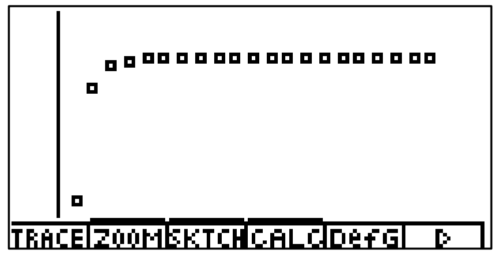
A scatterplot provides an interesting view of the data. From the MAIN MENU, choose STAT.

- Press [CTRL] [F3] to check the set up. Make sure the StatWind is set to automatic, pressing [F1] if necessary. Press [ESC] to return.
- We now want to delete any data in Lists 1 and 2. Press [F6] for more options, and with the cursor someplace in the list, press [F4] DEL-A to delete all of the items and [EXE] to confirm the deletion. Be sure to do this for Lists 1 and 2, and the other lists if you choose.
- Enter the number of days into List 1 and the amount of medication into List 2, pressing [EXE] after you type in each number.
- We'll now check the graph set up. Press [F6] for more options if necessary so that pressing [F1] gives the graph option and then press [5] for set.
- Use the down arrow to highlight GraphType. If needed press [F1] to make GraphType a scatterplot. Press the down arrow and make sure the Xlist is List 1 (pressing [F1] , [1] , then [EXE] if necessary when Xlist is highlighted) , the

Ylist is List 2 (following same procedure as with Xlist, except type 2 when Ylist is highlighted), and the frequency is 1 (pressing [F1] if needed when Freq is highlighted). See below left.

- Press [ESC] to return. Press [F1], [1] to view the scatterplot. See below right.





This graph should confirm for students that, though the amount of medication in the bloodstream may indeed be increasing, the amount of increase is insignificant after only a couple of days.

B. If you forget to take your medicine one day, you could either skip the day or take a double dose. Explore the different amounts in your bloodstream for each of these two options. Then determine which you think is better. Why?

Students should discuss this question before analyzing it. The teacher may try to have them think in terms of part A as they do so.

Let's suppose you forget to take your medicine the third day. The question becomes, should you just forget about it or should take a double dose on the fourth day. Let's look what happens if you skip the day.

On the second day, the amount of medicine in your bloodstream became 240 milligrams. If you forget on the third day, then the amount of medication in your bloodstream will be reduced by 80%. From the MAIN MENU, choose RUN-MAT.

- Type in 240 and press [EXE] .
- Press the subtraction key (remember, the calculator assumes you will be subtracting from the previous result), .8 (decimal for 80%), the multiplication key, [SHIFT] [(-)] for the Ans key, and [EXE] . See below left.

- Now we'll repeat what we did originally. Once again, press the subtraction key (remember, the calculator assumes you will be subtracting from the previous result), .8, the multiplication key, [SHIFT] [(-)] for the Ans key, but now also add 200. Then press [EXE] . See below right.

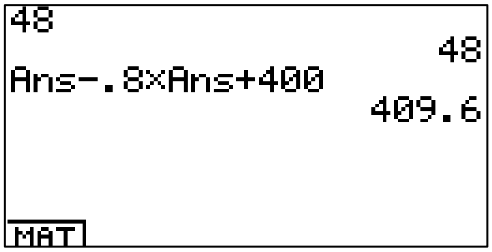
This means that after you take your pill on the 4th day, you will have 209.6 milligrams of medication in your system. If you continue to press [EXE] , you should see that the bloodstream again approaches 250 milligrams very quickly. You had the one bad day, one with little medication in your bloodstream, but you returned fairly quickly to the normal level.

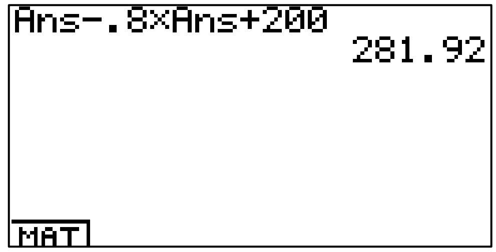
Now let's see what happens if you double up on the fourth day.

At the beginning of the third day, we know you have 48 milligrams in your bloodstream. To represent this, type in 48 and press [EXE] .

Now on the fourth day, you will have lost eighty percent of the 48 milligrams, but this time you will take 400 milligrams of medication. Press the subtraction key, .8, the multiplication key, [SHIFT] [(-)] (for Ans), the addition key, 400, and [EXE] . At this point you will have 409.6 milligrams of medication in your system. See below left.

We'll now return to the normal routine of only adding 200 milligrams into your bloodstream. Press the left cursor movement key and continue to press it until the 4 in 400 is highlighted. Change this to a 2 and the use the [DEL] key to remove the 4, then press [EXE] . See below right.





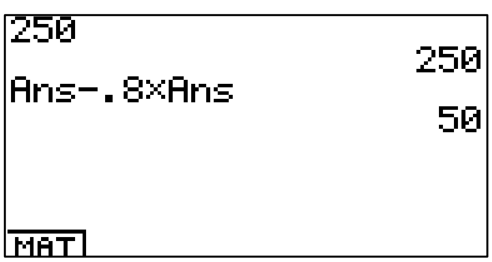
- Continue to press EXE to see what happens over time. Once again the amount approaches 250 milligrams, but this time the amounts are decreasing.

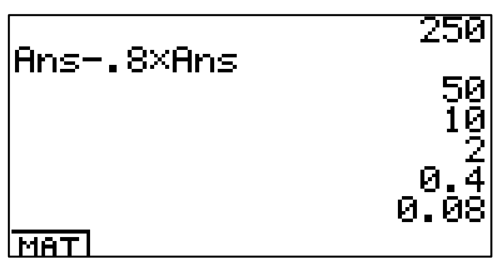
Should we skip the day or double the dose? We might think about what our bodies can tolerate. Putting too much medication into the bloodstream can be extremely harmful. Since the medication returns to the 250-milligram level quickly even after missing a day, just skipping the day will probably do little damage. On the other hand, doubling the dose for a single day has the potential for disaster.

C. How long does it take the medicine to be completely gone from your bloodstream? Support your answer.

After thinking about this, choose RUN-MAT from the MAIN MENU.

- Seed the calculator with 250 by typing in 250 and pressing EXE .
- Press the subtraction symbol, .8, the multiplication symbol, SHIFT , (-) and EXE . See below left.
- Continue to press EXE to see what happens over time. See below right.





These results show that, because 80% of the amount in your bloodstream is depleted each day, the number of milligrams remaining becomes extremely small very quickly. In theory, the result would never reach 0; you are always retaining 20% of what's in your system, and 20% of something greater than 0 is always greater than 0. At some point, however, our mathematical model necessarily fails – we can't go below the molecular level! An interesting exploration might be to conduct some research to see how long traces of certain medications actually remain in the bloodstream.

PROBLEM 3: INVESTING MONEY

Suppose you have been given $1,000, but you must invest it for 20 years. You could leave it in the bank and earn 2% interest each year. You could put it in a money market and expect to make 6% interest each year. You could also put the money into the stock market, where your money might grow more quickly, but you also run the risk of losing it. Despite the risks, however, you are told that in one fund you can expect to earn 10% on your money each year, and in another, you might be able to earn 14% each year.

A. First, estimate how much you think your investments will be worth in 20 years.

2%: ____________ 6%: _____________ 10%: _____________ 14%: _____________

B. Complete the table below showing how much your investment is worth.

YEAR	**@2%**	**@6%**	**@10%**	**@14%**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

C. Compare these amounts with what you would have if you invest $5,000, but only for 5 years.

PROBLEM 4: PENNY OR $1000?

Suppose you have a choice of two payment plans for a special job on which you will work thirty days. For your first choice, you will earn $.01 the first day, $.02 the second day, $.04 the third day, $.08 the fourth day, and so on throughout the days. The second plan offers you $1,000 the first day; $2,000 the second day; $3,000 the third day; $4,000 the fourth day; and so on through the month. Which plan should you select? After you make your choice, determine how much difference there is between the two plans.

MATERIALS

Casio *Algebra FX 2.0* Graphing Calculator

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AW – Foundations of Algebra and Geometry (1998)	1.3; 2.3; 3.3; 4.1; 6.1; 8.1-3
Glencoe – Mathematics Applications and Connections C1 (1995)	2.6; 3.10; 8.5; 10.5-6; 10.8; 13.5
Glencoe – Mathematics Applications and Connections C2 (1995)	3.7; 4.3; 6.4; 7.6; 11.7-9; 12.1; 14.3-4
Houghton Mifflin – The Mathematics Experience I (1992)	10.3; 10.7-10; 10.14-15
Houghton Mifflin – The Mathematics Experience II (1992)	1.10; 3.11-12; 4.5; 5.13; 12.2-3; 15.5; 15.8
McDougal Littell – Gateways to Algebra and Geometry (1994)	2.4; 5.3; 9.7; 12.1-2
Prentice Hall – Middle Grades Mathematics C1 (1995)	1.7; 5.4; 5.6-7; 9.8; 9.10
Prentice Hall – Middle Grades Mathematics C2 (1995)	1.5; 3.10; 6.7; 9.7-11; 11.5
Prentice Hall – Middle Grades Mathematics C3 (1995)	1.6; 4.7; 5.2-3; 5.5; 6.1-3; 6.7; 10.1; 10.4; 10.6; 10.8
SFAW – Middle School Math C1, V1 (1999)	1.3; 2.11
SFAW – Middle School Math C1, V2 (1999)	
SFAW – Middle School Math C2, V1 (1999)	1.5-6; 2.1; 2.3; 2.5
SFAW – Middle School Math C2, V2 (1999)	8.7; 10.2-5
SFAW – Middle School Math C3, V1 (1999)	1.7; 3.1-3; 4.1; 4.3; 6.2; 6.4-6
SFAW – Middle School Math C3, V2 (1999)	10.1-2
SFAW: UCSMP – Transition Mathematics, Part 1 (1998)	2.5-6;; 4.2-3; 4.7
SFAW: UCSMP – Transition Mathematics, Part 2 (1998)	8.4; 13.1