

## CLEMSON ALGEBRA PROJECT UNIT 1: EVALUATING EXPRESSIONS

### ***PROBLEM 1: SUMMER EMPLOYMENT***

This summer you will be working as a lifeguard at the city pool. Your rate of pay will be \$8.26 per hour.

- A. Write an algebraic expression that describes your gross pay after working  $h$  hours.
- B. What does the coefficient of  $h$  represent?
- C. Using the expression from part A, determine how many hours you must work in order to earn at least \$300.00 per week.
- D. Upon examining your paycheck stub, you notice that after working 40 hours, you paid \$25.03 Social Security Tax. Write an expression to determine the percent of your gross pay that you paid for Social Security Tax. Define any variable you use.
- E. What is the percentage of Social Security Tax that you pay?

### ***MATERIALS***

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

### ***EXTENSIONS***

1. Have students compare the earnings from two or more prospective jobs.
2. Have students bring in actual paycheck stubs and calculate the actual percentages for all withholdings.

## EXPRESSIONS

### **ONE SOLUTION TO PROBLEM 1: SUMMER EMPLOYMENT**

**A. Write an algebraic expression that describes your gross pay after working  $h$  hours.**

Simply multiply the amount you earn each hour by the number of hours you work. Notice that  $\frac{\text{dollars}}{\text{hour}} * \text{hours} = \text{dollars}$ . Here our answer is \$8.26 per hour times  $h$  hours, or  $8.26h$ . Our answer will be in dollars.

**B. What does the coefficient of  $h$  represent?**

The coefficient of  $h$ , in this case 8.26, represents the hourly rate of pay. Here this means you earn \$8.26 every hour.

**C. Using the expression from problem 1, determine how many hours you must work in order to earn at least \$300.00 per week.**

One way to approach this problem is with a table of values. From the MAIN MENU on your calculator:

- x Call up the “Table” menu.
- x Press **SHIFT** **MENU** for SET UP. Make sure the option for “Variable” is set to “Range.” Press **F1** if needed, then **EXIT**.
- x We will use  $x$  instead of  $h$ . After Y1, type **8** **.** **2** **6** **X, θ, T** and **EXE**. See below left.

Our next step is to set the Range. Students should discuss the values for  $x$  which are of interest. Because the hourly rate may change after 40 hours (perhaps you receive 1.5 times your regular rate for any overtime hours), you may wish to set the upper limit at 40. To set the range:

- x Press **F5** to access the RANGE screen.
- x Set Start and End values as appropriate. Press **EXE** after each entry.
- x The pitch indicates how to increment  $x$ . One reasonable value is .5. See below right for one possibility.

## EXPRESSIONS

Table Func :Y=
Y1: 8.26X
Y2:
Y3:
Y4:
Y5:
Y6:
SEL DEL TYPE COLS RANG TABEL

Table Range
X
Start: 0
End : 40
Pitch: 0.5

- x When all values have been input, press **EXIT** and then **F6** to see the table. See below left for the beginning of the table.

Press the down arrow until you see 36.5 in the X column. The corresponding value in the Y1 column is 301.49. Students should conclude that one must work between 36 and  $36\frac{1}{2}$  hours in order to earn at least \$300.00 per week. Practically speaking, they would have to work either  $36\frac{1}{2}$  or 37 hours, depending on the time periods used by the employer.

X	Y1
0	0
0.5	4.13
1	8.26
1.5	12.39

0

FORM DEL ROW G·CON G·FLT

X	Y1
35.5	293.23
36	297.36
36.5	301.49
37	305.62

37

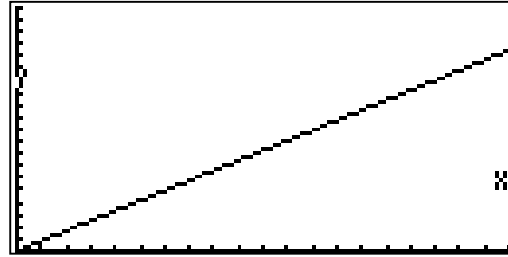
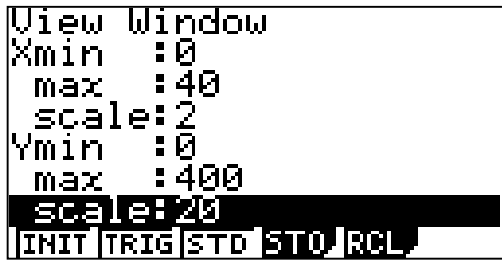
FORM DEL ROW G·CON G·FLT

If we were not restricted to half-hour or hour increments, we may wish to get a more precise answer. One way to do so is with a graph. To do this:

From the MAIN MENU, select “Graph.”

- x The expression should already be entered.
- x Press **SHIFT** **F3** to check the viewing window.
- x Use the information from the table to set the domain and range, pressing **EXE** after each entry. See below left for a possible set of values.
- x After all entries have been made, press **EXIT** and then **F6** to draw the graph. See below right.

## EXPRESSIONS



The calculator allows us to find points very easily. With the graph shown,

- x Press **F5** to access the graph solver.
- x Press **F6** for more options.
- x Press **F2** for “X-CAL.”
- x Type 300 after the Y = and press **EXE** .

The calculator moves to the requested point, which tells us that we must work 36.31961259 or approximately 36.3 hours to earn \$300.00.

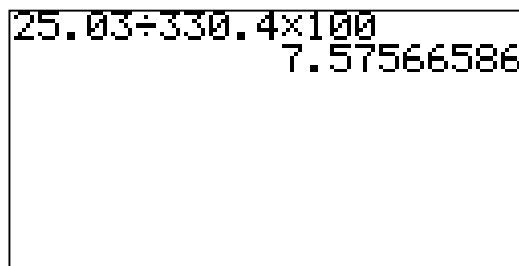
**D. Upon examining your pay check stub, you notice that after working 40 hours, you paid \$25.03 Social Security Tax. Write an expression to determine the percent this tax is of your gross pay. Define any variables you use.**

Quite simply, all we need do is compute  $\frac{\text{Tax}}{\text{GrossPay}} * 100$  to find this percent.

**E. What is the percentage of Social Security Tax that you pay?**

First, we need to determine the gross pay for 40 hours. To do this, we could scroll down through the table. Alternately, we could use the “Y-CAL” feature on the graph solver. In either case, we should determine that the gross pay is \$330.40

To perform the final calculation, select “Run” from the MAIN MENU. Enter the values as shown below and press **EXE** . Our result is 7.58%.



## EXPRESSIONS

### ***PROBLEM 2: COMMUNITY SERVICE***

The Community Service Club at your school wants to provide fresh vegetables to the community soup kitchen. Club members decide to plant a vegetable garden this spring. The shape of the garden will be a rectangle. A local hardware store has donated 100 feet of fencing for you to use to enclose the garden. Club members must determine the dimensions of the garden that will give the maximum area for planting the garden.

- A. Write an expression that expresses the length of the rectangle in terms of the width.
- B. Write the formula to find the area of a rectangle.
- C. Rewrite this formula using the expression from problem A.
- D. Find the width of the rectangle that gives the maximum area.
- E. Find the length of the rectangle.
- F. Describe the rectangle.

### ***EXTENSION***

Select other values for the perimeter of the garden. Make a conjecture about the type of rectangle that results in the maximum area for a given perimeter.

EXPRESSIONS

**ONE SOLUTION TO PROBLEM 2: COMMUNITY SERVICE**

**A. Write an expression that expresses the length of the rectangle in terms of the width.**

If we let  $l$  represent the length and  $w$  the width, we know that  $2l + 2w = 100$ . To

solve for  $l$ , we must subtract  $2w$  and divide by 2, obtaining  $l = \frac{100 - 2w}{2}$ .

**B. Write the formula to find the area of a rectangle.**

Letting  $A$  represent the area, we have the familiar  $A = l * w$ .

**C. Rewrite this formula using the expression from problem A.**

Substituting our expression from part A into part B, we obtain

$$A = \frac{100 - 2w}{2} * w.$$

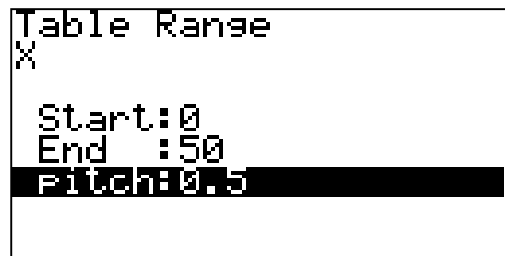
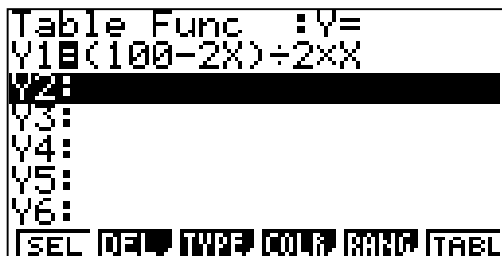
**D. Find the width of the rectangle that gives the maximum area.**

There are several ways to go about this, but we will again choose to use the table. From the MAIN MENU, choose the option for "Table." Then,

- x Enter the formula in the Y1 position, using  $x$  instead of  $w$ . Make sure parentheses are placed around the numerator as shown below left. Press EXE after you have finished.

To set the range,

- x Press F5.
- x Type in appropriate values, pressing EXE after each entry. See below right. When finished press EXIT.



## EXPRESSIONS

- x To see the table, press **F6** . Use the down and up arrow keys to move through the table.

As you scroll through the table, you are looking for the X value (the width) that produces the greatest Y1 value (the area). The highest value that we see is 625, which is produced when X is 25. See the screen below. This tells us that a width of 25 feet yields the greatest area, which is 625 square feet.

X	Y1
24	624
24.5	624.75
25	625
25.5	624.75

25.5
FORM DEL ROW
G-COM G-FLT

Instead of a table, we could instead use a graph. From the MAIN MENU, select the “Graph” option. The function should already be in the Y1 position. Then,

- x Press **SHIFT** **F3** to access the window.
- x Type in reasonable values, pressing **EXE** after each entry. See below left for one possible screen. When finished, press **EXIT** .
- x To draw the graph, press **F6** .
- x Now press **F5** to access the graph solver.
- x Press **F2** to find the maximum, which again is (25, 625). See below right.

```

View Window
Xmin : 0
max  : 50
scale: 5
Ymin : 0
max  : 1000
scale: 50
INIT TRIG STD STO RCL
    
```

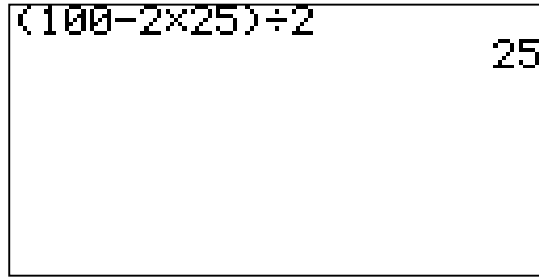


## EXPRESSIONS

### E. Find the length of the rectangle.

Earlier we determined the relationship between the length and the width to be  $l = \frac{100 - 2w}{2}$ . All we need do is determine the length we need when the width is 25.

To do so, from the MAIN MENU, choose “Run.” Simply type in the values as shown below and press **EXE**. This tells us that the length should be 25 feet also.

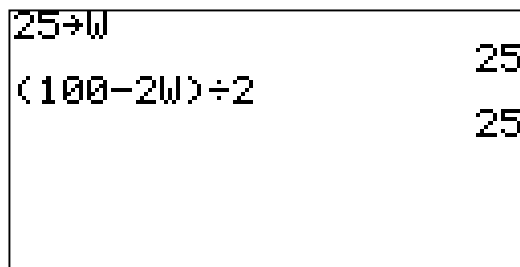


```
(100-2*25)÷2
25
```

### F. Describe the rectangle.

Because the length and the width are the same, both 25 feet, the rectangle that gives the maximum area is a square.

Another way to evaluate expressions on the graphing calculator is available with the “Run” menu. Using the storage key (the right arrow key) and the alpha keys, we can store values for variables and then type in any expression in which we are interested. For example, in our problem above we could store 25 in for  $W$  and then use  $W$  in an expression. See the screen below.



```
25→W
(100-2W)÷2
25
25
```

## EXPRESSIONS

### ***PROBLEM 3: COLLEGE VISIT***

You are planning to visit Lehigh University in Bethlehem, PA. Because your entire family is going, you plan to drive. After consulting a Road Atlas, you determine that the distance between Columbia, SC, and Bethlehem, PA, is 750 miles.

- A. If your average speed is 62.5 miles per hour, how long can you expect to be driving?
- B. If you plan to take a break every  $3\frac{1}{2}$  hours, how far can you expect to travel between breaks?

### ***EXTENSION***

Select a college or university that your students may be interested in visiting. Have students determine the distance between your town and the college or university and the amount of time it will take to make the trip.

### ***PROBLEM 4: CLASS PLAY***

The freshman class is planning its annual play. Production expenses were high and you determine that the revenue from ticket sales must be at least \$2,000.00. If you charge \$5.00 admission for every adult and \$2.50 for every student, determine the combinations of adult tickets and student tickets that must be sold to produce the \$2,000.00.

## EXPRESSIONS

### TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	2.2
AWSM – Focus on Advanced Algebra (1998)	
Glencoe – Algebra 1 (1998)	1.1
Glencoe – Algebra 2 (1998)	1.1, 1.5
Holt Rinehart Winston – Algebra (1997)	1.3, 1.4
Holt Rinehart Winston – Advanced Algebra (1997)	
Key Curriculum – Advanced Algebra Through Data Exploration	
Merrill – Algebra 1 (1995)	1.2
Merrill – Algebra 2 (1995)	1.3
McDougal Littell – Algebra 1: Explorations and Applications (1998)	
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	1.2, 1.3, 1.4, 2.1
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	1.8
Prentice Hall – Algebra (1998)	1.2, 1.3, 1.6, 1.9, 2.4, 2.5
Prentice Hall – Advanced Algebra (1998)	1.7
SFAW: UCSMP – Algebra Part 1 (1998)	1.4, 4.1, 5.2, 5.4
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	2.2