CLEMSON ALGEBRA PROJECT
UNIT 10: RATIONAL EXPRESSIONS AND FUNCTIONS

PROBLEM 1: LIGHT INTENSITY

Light intensity is important in many aspects of our lives. For example, stairwells in public buildings, dark city streets, and classrooms must be properly illuminated. Sailors rely on the illumination from lighthouses in coastal regions. This investigation simulates determining the level of light intensity from a single point source such as a lighthouse or a lamp in a stairwell.

A. Using a light source, a light probe, and a data collector, gather data on the intensity of the light emanating from the light source at several distances, using distance as the independent variable.

B. Construct a scatterplot of the data. Describe the shape of the graph.

C. Are there any asymptotes? If so, estimate the value of these asymptotes. Explain this asymptotic behavior.

D. Calculate a mathematical model that describes the data. How well does the model fit the data?

E. Use your model to predict the light intensity at 6 feet from the light source and at 10 feet from the light source.

F. In theory, light intensity conforms to the model $I = \frac{k}{d^2}$, where I is the intensity of the light, k is a constant, and d is the distance from the light source. How closely does your mathematical model match this model? Explain any differences.

MATERIALS

Light Source (Flashlight without lens, Trouble Light, or Lamp without shade.)

Tape measure

Meter Stick (Optional)

Casio EA-100 Data Collector

Light Probe

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

EXTENSION

Suppose the point source emits light equally in all directions and is placed at the midpoint of the tape measure that you placed on the floor. Describe the graph that you predict would be generated by the data collected if measures were taken from both sides of the light source. See diagram below. If possible, conduct the experiment to verify your answer.

ONE SOLUTION TO PROBLEM 1: LIGHT INTENSITY

A. Using a light source, a light probe, and a data collector, gather data on the intensity of the light emanating from the light source at several distances, using distance as the independent variable.

The following procedures can be used to gather the data.

1. One student will hold the light source, one student will collect data using the EA-100, and a third student will record the distance from the light source and the light intensity measurements from the data collector.
2. Open the tape measure and place it on the floor.
3. Insert the light probe into the channel 1 port on the data collector.
4. Turn on the data collector. "Rec Time" should be blinking and DONE should be displayed on the left side of the viewing screen.
5. Press [MODE] . This puts the EA-100 in multimeter mode, allowing you to read data when you wish.
6. Collect your data. To do so, point the front end of the light probe at the light source. Get close to the light source until -Hi- is visible in the viewing screen. Move slowly away from the light source until a numerical reading is visible in the viewing screen. Record this number and the distance from the light source. Move slowly away from the light source, **keeping the light probe at the same height**. It may be necessary to stabilize the light probe by holding it tightly against a meter stick in order to maintain the same height. You may actually use masking tape to affix the light probe to the meter stick to maintain the light probe at the same height. Make several recordings of the distance from the light source and the light intensity.

The solution presented here uses data from a previous experiment. The data, and thus the actual student answers, will vary from those shown below.

Distance from Light Source in feet	Light Intensity
1	791
1.5	478
2	351
2.5	293
3	232
3.5	192
4	157

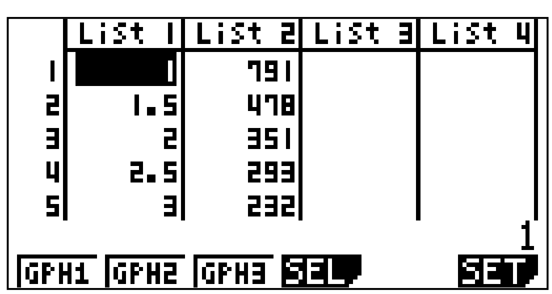
B. Construct a scatterplot of the data. Describe the shape of the graph.

From the MAIN MENU, call up "Statistics." Then,

x Clear the data in List 1 if necessary by pressing [F6], [F4], and [F1]. Use the right cursor arrow to move the cursor to List 2. To clear the data in List 2, again press [F4] and [F1].

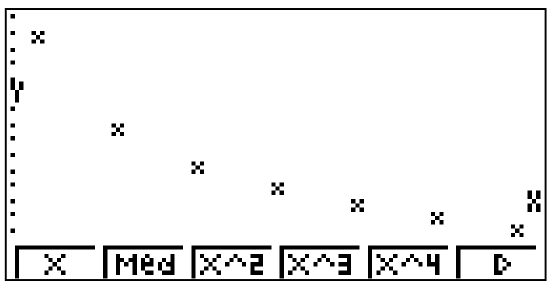
x Press [SHIFT] [MENU] to check the set up. "Stat Wind "should be set on Auto. If it isn't, press [F1]. Return to the lists by pressing [EXIT].

x Enter the data from the table into Lists 1 and 2 pressing [EXE] after each entry. See below for the beginning of the lists.



We're now ready to construct the scatterplot. If necessary, press F6 so that "GRAPH" is listed as a function option on the screen.

- Press F1 to access the graph option.
- Check the graph SET UP by pressing F6 .
- Make sure "Statgraph1" is listed. Then use the down arrow and highlight "Graph Type." Press F1 for scatterplot. Make sure that the Xlist is List 1 the Ylist is List 2, and the frequency is 1. Choose other options as you prefer. Press EXIT to return to the lists.
- To graph the scatterplot, press F1 for graph 1. See below.



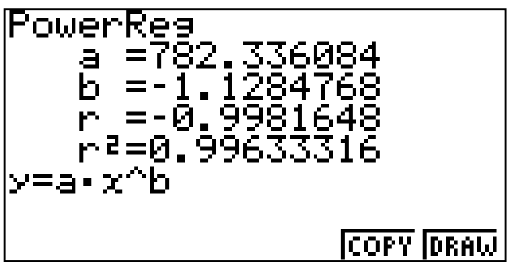
The graph shows the light intensity decreasing, but the plot is not linear. It appears that the light intensity is decreasing, but not at a consistent rate. It decreases at a slower rate as the distance from the light source becomes greater.

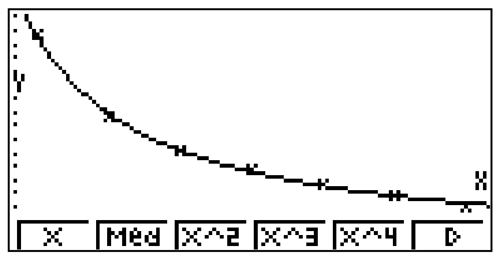
C. Are there any asymptotes? If so, estimate the value of these asymptotes. Explain this asymptotic behavior.

There do appear to be asymptotes, including a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. The vertical asymptote occurs because the light intensity increases at ever increasing rates as we get closer and closer to the source. The horizontal asymptote occurs because as we get farther and farther away from the light source, the intensity of the light approaches, but theoretically at least, does not quite reach 0 at ever decreasing rates.

D. Calculate a mathematical model that describes the data. How well does the model fit the data?

- From the screen displaying the scatter plot, press F6 for more options.
- Because light intensity is modeled by an inverse square law, press F3 for power regression. See below left.
- Copy this function rule into Y1 so that you can access the function later. To do so, press F5 . To store the equation in the high lighted position press EXE .
- To superimpose the graph of the regression equation over the scatterplot press F6 . See below right.





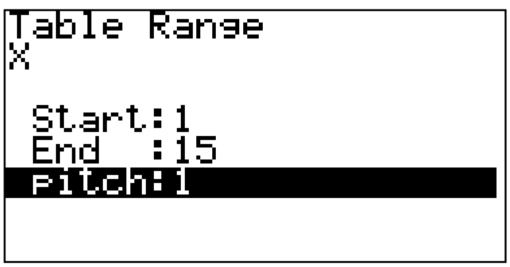
The regression equation obtained is $y = 782.3 * x^{-1.128}$. If the model from the data had matched the theory, the value for b would have been –2. Students may experiment with other regression models to find a regression equation that fits better. By saving the equations, students may change color and superimpose several models over the scatterplot simultaneously to help determine which model best fits the data. However, caution them about searching for models for which they do not have some rationale. Without some underlying theory, complex models may fit the sample data, but probably cannot be generalized. Theoretically, an inverse squared function should best fit the data.

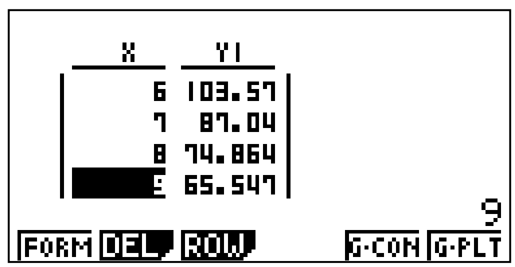
E. Use your model to predict the light intensity at 6 feet from the light source and at 10 feet from the light source.

From the MAIN MENU, select "TABLE."

- If you have completed the steps listed above, the regression equation is already stored in Y1. Delete or de-select any other functions.
- To set the Range for the table, press [F5] . Start at 1 and end at 15 with a pitch of 1, remembering to press [EXE] after each entry. See below left.
- Press [EXIT] to return to the previous screen and [F6] to see the table.

Use the arrow keys to find the intensity at distances of 6 and 10 feet. Your answers should be 103.57 and 58.199, respectively. See below right for the solution when the distance is 6.





F. In theory, light intensity conforms to the model $I = \frac{k}{d^2}$, where *I* is the intensity of the light, *k* is a constant and *d* is the distance from the light source. How closely does your mathematical model match this model? Explain any differences.

The experimental model is different from the theoretical model as mentioned above. Several sources of error in the experimental model are possible. Students may make errors in determining the distance from the light source. More importantly, background lighting will likely cause errors. Also, limitations of the measuring devices themselves can affect the measurements.

PROBLEM 2: DEER POPULATION

Because of limitations in habitat and food sources, populations do not typically grow exponentially. Suppose that, based on experience, officials estimate that the size of a herd of deer t years from now can be estimated by the expression $\frac{10(5+3t)}{1+0.04t}$.

A. What is the initial population of deer?

B. How long does it take the initial population to double? Explain.

C. Find the size of the herd after 5 years, after 10 years, and after 50 years.

D. When does the rate of growth of the deer population appear to be greatest? Why do you think the rate of growth of the deer herd slows down? Explain your reasoning.

E. What appears to be the maximum number of deer this area can support? Explain your reasoning.

REFERENCE: *Precalculus, Third Edition,* by Larson and Hostetler, D.C. Heath and Company, 1993.

ONE SOLUTION TO PROBLEM 2: DEER POPULATION

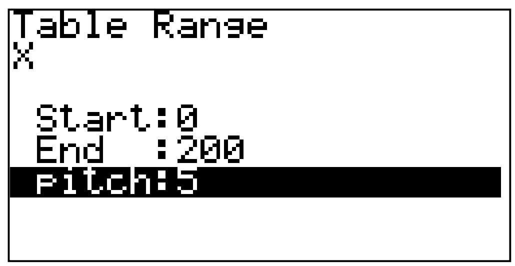
A. **What is the initial population of deer?**

This problem can be solved in several ways. In this solution, a table is used. From the MAIN MENU, select "Table." Then,

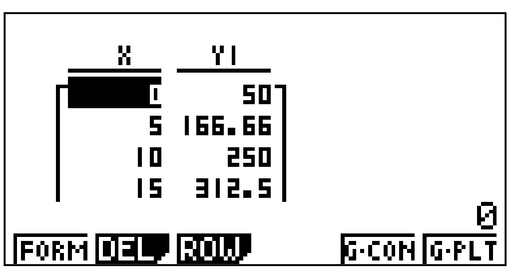
x Clear any functions by highlighting them and pressing [F2] and [F1] . Alternatively, de-select them by highlighting them and pressing [F1] .

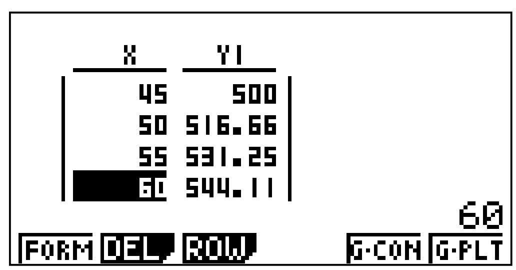
x Type in the formula and press [EXE] . See below left.

x Press [F5] to set the range of the table and view the table. You may wish to investigate from 0 to 200 years, incrementing by 5 years, as shown below right. Press [EXE] after each entry.



x To return to the previous screen, press [EXIT] . Press [F6] to see the table.





From the table we can see that $y = 50$ when $x = 0$. Therefore, the initial deer population was 50 deer.

B. How long does it take the initial population to double? Explain.

Upon examination of the table in part A, one notices that the doubling time is occurs between year 1 and year 5. To get a closer estimate of the doubling time, reset the pitch for the range.

- Type [EXIT] and [F5] . Change the pitch to 1.
- Press [EXIT] to return to the table menu.

At $x = 2$, $y = 101.85$, therefore the student might conclude that the doubling time for the deer population is somewhat less than 2 years.

A more precise answer is found by using the Equation Solver.

- Type [MENU] [X, θ, T] .
- Select the Solver by typing [F3] .
- Enter the equation $10(5 + 3x) \div (1 + .04x) = 100$.
- To solve the equation, type [F6] . You find that the doubling time is approximately 1.92 years.

C. Find the size of the herd after 5 years, after 10 years, and after 50 years.

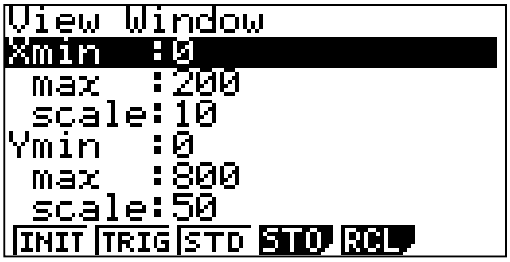
After 5 years there are approximately 166 deer in the herd, after 10 years there are approximately 250 deer in the herd, and after 50 years there are approximately 516 deer in the herd. Note that at the beginning of this study, there were 50 deer. See the screen on page 9.

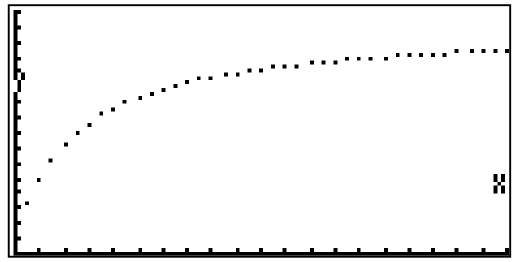
D. When does the rate of growth of the deer population appear to be greatest? Why do you think the rate of growth of the deer herd slows down? Explain your reasoning.

We'll draw a scatterplot of the data to investigate this.

- First, check the viewing window by pressing [SHIFT] [F3] . Based on the table information, one possible window is shown below left. Press [EXE] after each entry and [EXIT] when finished.

- Press F6 to return to the table and F6 again to create the scatterplot. See below right. Press F1 and the arrow keys to trace through the points.





The rate of growth appears to be greatest between 0 and 50 years, as the slope of the curve is greatest between these years. One possible explanation for a slowing in the rate of growth is that the amount of food available to each animal decreases as the population increases.

E. What appears to be the maximum number of deer this area can support? Explain your reasoning.

The maximum number of deer that can be supported in this area appears to be approximately 670. The curve flattens out and becomes almost horizontal indicating a slope nearing zero, indicating little to no growth. In other words, the function appears to have a horizontal asymptote. Either by exploring a greater domain or by computing the algebra, this asymptote is actually at 750 deer. Of course, many factors will affect the deer population over such a long time period.

PROBLEM 3: STRESS TEST

Ionization meters measure radiation emitting from a mass in millirems per hour (mR/hr). These meters are often used to check for radiation leaks in industrial facilities that use irradiation during the manufacturing process. Recently during a radiation safety check, a technician noticed that when one of the plant workers walked past the meter, readings became higher than normal and when the worker walked away, the readings became much lower. The presence of this worker was disrupting the safety inspection.

After talking with the worker, the technician learned that this worker had just undergone a stress test during which radioactive isotopes were injected into her to monitor blood flow to the heart. They decided to determine how far away the worker had to stand so that the safety inspection could proceed without interruption.

They set up the following investigation. A tape measure was extended on the floor. The technician held the ionization meter and the worker walked away from the meter. The technician recorded the radiation emitted. The results are shown below.

Distance from Meter in Feet	MR/hr
1	2
2	.95
3	.56
4	.33
5	.22
6	.17
7	.15
8	.11
9	.07
10	.07
11	.07
12	.04
13	.04
14	.04

A. Construct a scatterplot of the data. Describe the graph.

B. Write an equation that describes the decrease in radiation measured as the worker walks away from the ionization meter.

C. How far must the worker walk before the ionization meter registers 0 mR/hr? Justify your reasoning.

PROBLEM 4: SEESAW

At recess Johnny and his friends like to play on the seesaw. They like to take turns, and one person always gets to stay on the seesaw for the entire recess. Today is Johnny's turn to stay on for the entire recess. Johnny weighs 75 pounds and always sits 5 feet from the center of the seesaw. Assuming they keep the seesaw balanced, how much does each of his friends weigh if they sit 3 feet from the center, 3.5 feet from the center, 4 feet from the center, 4.5 feet from the center, 5 feet from the center, and 6 feet from the center, respectively?

Distance from the center of the see-saw	Weight
3 feet	
3.5 feet	
4 feet	
4.5 feet	
5 feet	
6 feet	

EXTENSION

Could someone who weighs 130 pounds go on the seesaw with Johnny if Johnny has to stay five feet from the center? Explain your reasoning.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	10.1
AWSM – Focus on Advanced Algebra (1998)	7.1
Glencoe – Algebra 1 (1998)	4.8, 12.8
Glencoe – Algebra 2 (1998)	9.1, 9.2
Holt Rinehart Winston – Algebra (1997)	2.3, 14.1, 14.2
Holt Rinehart Winston – Advanced Algebra (1997)	9.1, 9.2
Key Curriculum – Advanced Algebra Through Data Exploration	12.1, 12.2
Merrill – Algebra 1 (1995)	4.9
Merrill – Algebra 2 (1995)	11.2
McDougal Littell – Algebra 1: Explorations and Applications (1998)	11.1
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	11.3, 12.5
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	8.10, 12.7
Prentice Hall – Algebra (1998)	11.1, 11.2, 11.5
Prentice Hall – Advanced Algebra (1998)	8.1, 8.2
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	2.2, 2.6, 2.7, 2.8
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	14.1, 14.2