CLEMSON ALGEBRA PROJECT
UNIT 11: QUADRATIC MODELS

PROBLEM 1: JAR ROLL

Set a table on an angle. Take a cylindrical can or jar and push it so it rolls up and then back down the table. You are to investigate the object's position on the table over time.

A. Determine and calculate an appropriate model.
B. Investigate the errors in the model.
C. Explore the differences in successive distance measurements. Make a conjecture as to what these differences represent, and model them mathematically.
D. Finally explore the second order differences. Make conjectures as to what these differences, and model them mathematically.

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

QV780 Digital Camera and Meter Stick

Alternate to QV780 and Meter Stick: EA-100 Data Collector and Pasco Motion Detector

A cylindrical jar or can

EXTENSIONS

1. How does the model change if you move the motion detector or meter stick up or down a few centimeters?
2. What happens if you put the motion detector on the top of the table or reverse the meter stick, putting 0 at the top instead of the bottom?
3. What happens as you change the angle of the table?
4. Investigate the changes that occur if you roll different objects.
5. Investigate the changes that occur as you change the force with which you push the object up the incline.

ONE SOLUTION TO PROBLEM 1: JAR ROLL

A. Determine and calculate an appropriate model.

1) Make a conjecture as to which model is appropriate. Discuss the pros and cons.

Due to gravity, the object will accelerate in the downward direction. Because the rate at which the object picks up speed is, at least theoretically, constant, the appropriate model is quadratic.

Students, of course, may have difficulty determining an appropriate model and may wish to look at a scatterplot before making a conjecture as to the type of mathematical model they should use. Although this is certainly a natural, intuitive approach, caution them that if they do not have a rationale for a particular model, they should conduct several experiments with as many data points as possible to verify that their model is appropriate. Without obtaining this confirmation, the model they have chosen may fit their particular sample data, but may not generalize to other similar situations.

Similarly, even though their calculators can perform very complex regressions, such as quartic, higher degree models require both a theoretical justification and an abundance of data points to have any validity. To convince yourself of this, make up any data for, say, six points. Calculate the quartic regression model and observe how well it fits your data. With only a few data points, complex models are too sample specific to be of much use in the real world. Simply put, they should not be generalized.

2) Gather the data with the QV-780 Digital Camera.

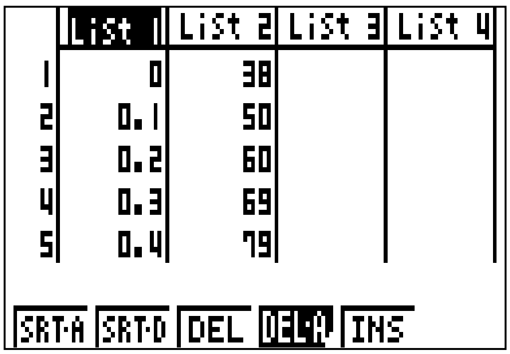
Prop up one side of a table or board that is at least a meter long. Attach a measuring tape or secure a meter stick to the table, with the 0 measure near the bottom. You will make a movie of the rolling object, so find a position above the table that allows you to see the entire meter stick. It will probably be necessary to make large marks near the meter stick so you can interpolate the measures more easily.

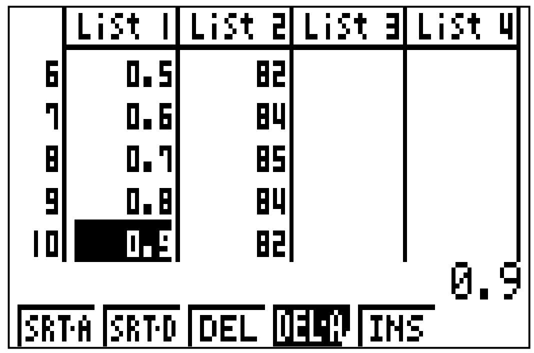
(NOTE: The data can be collected perhaps more easily with a motion detector, but giving students many opportunities to read and interpolate measuring devices is also worthwhile. Also getting the data collector to pick up only the jar can be problematic. As an alternate to the problem discussed, you may wish to use the data collector to chart the path of a free-falling object. The mathematics is fundamentally the same, but students will not see both “sides” of the parabola.)

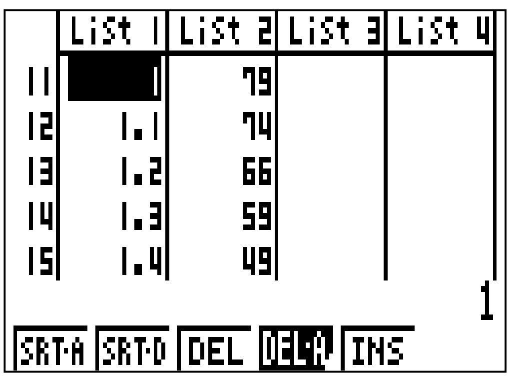
Practice giving the can or jar a shove so that you release it near the bottom of the measuring device, and it rolls up and back down the table, taking approximately two seconds to complete its roll. Try different objects to find one that rolls smoothly. You will probably also have to experiment with the angle of the table.

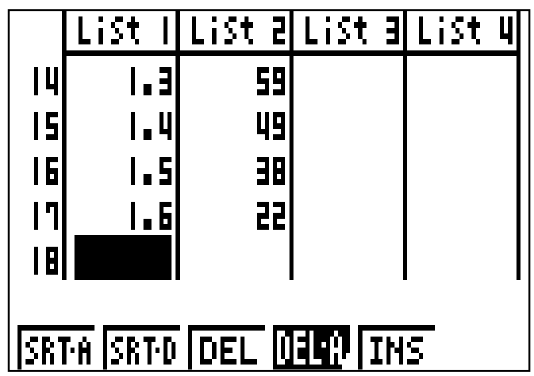
- Turn your digital camera on in RECord mode.
- Press MODE a sufficient number of times until you see the movie icon. Find a position so that you can see the entire roll.
- Working with your partner, make a movie of one complete roll by pressing and holding down the shutter. This may take a few attempts.
- Switch your camera to PLAY mode, and find the movie. Press the shutter at the start of the movie, then MENU to pause the action. Use the + key to advance the movie one frame at a time.
- Label any beginning point you observe during the roll 0. Set up a list of ordered pairs with x representing the frame number (beginning with 0) and y representing the measure on the meter stick. Construct the lists by advancing through the movie one frame at a time. Because the camera takes a picture every tenth of a second, count by .1 to build your x list, and use the observed location to build List 2. Be consistent with the part of the object you are looking for. Construct your lists for the duration of the roll. If the roll lasted two seconds, you will have 21 data points to use.
- We now need to enter the data into the calculator. Turn your calculator on. From the MAIN MENU, call up the “List” mode.

- If there are data in Lists 1 and 2, press [F4] followed by [F1] to delete all of the elements. Alternatively, press [SHIFT] [MENU] for the setup and press a function key to work in a different List File.
- In List 1, enter the times. For the experiment conducted in the development of the unit, the 17 data points are shown on the four screens below. Note that points 14 and 15 are shown twice.







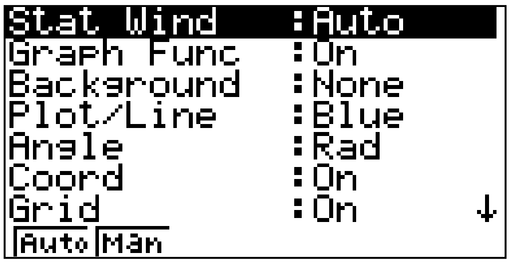


3) Construct a scatterplot, setting an appropriate window.

Before constructing the scatterplot, we'll make sure the window will be set automatically and that a list of residuals will be created automatically for us.

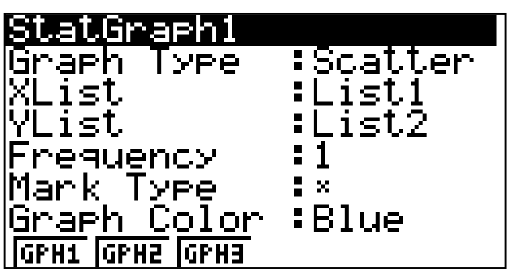
- From the MAIN MENU, choose "Statistics."
- Press [SHIFT] [MENU] to access the SET UP.
- Make sure the "Stat Window" is set on "Automatic." If necessary, with "Stat Window" highlighted, press [F1] . See below left.

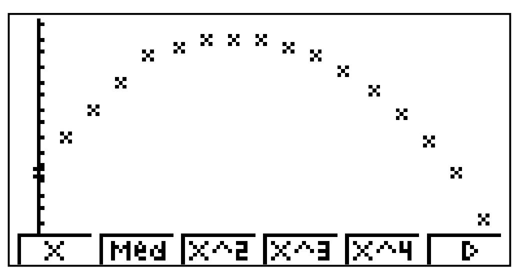
- Use the down arrow to highlight "Resid List."
- Press F2 for LIST.
- Press F3 to have residuals created automatically in List 3. See below right.
- Press EXIT to return to the primary statistics screen.



We're now ready to create the scatter plot.

- Press F1 to go to the GRPH menu, then F6 for SET.
- Set StatGraph1 as a Scatter graph, using List1 for the Xlist and List2 for the Ylist. The Frequency should be 1, and the Mark Type and Graph Color can be whatever you desire. See below left.
- When finished, press EXIT to return to the graph menu screen.
- Press F1 to see StatGraph1, the scatterplot. See below right.

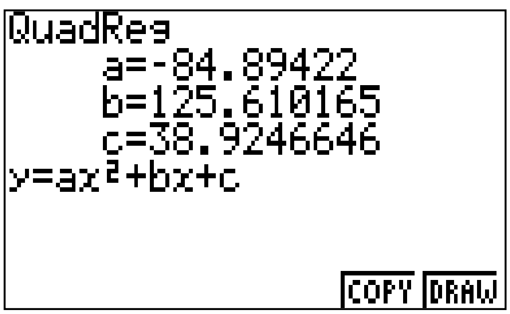


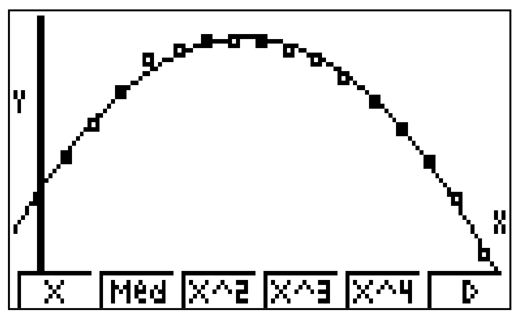


Your graph should look somewhat similar to the one above. By pressing SHIFT F1 , you can access the Trace function and use the right and left arrows to see the values of each of the data points. Pressing EXIT and F1 will return you the graph without the Trace function.

4) Calculate the regression model and graph the regression model on the scatterplot.

- With the scatterplot in view, the bottom menu gives several regression models. Because we are interested in a quadratic model, press F3 for the X^2, or second-degree regression. See the screen below left.
- Press F5 and EXE to copy the regression equation into Y1. Press F6 to draw the regression equation on your scatterplot, giving a visual representation of how well the mathematical model fits the data. This is shown in the figure on the right below.





5) ***Interpret the y-intercept and other features of the regression equation.***

Rounded to four significant digits, the regression equation is $y = -84.89x^2 + 125.6x + 38.92$. The *y*-intercept, the value obtained for *y* when *x* is 0, is 38.92. Too often in using models, students do not make sense of the variables, of putting real-world meaning to the variables. In the model here, *x* represents time, measured in seconds. (When making a movie, the camera snaps a picture every tenth of a second.) The *y*-value represents the distance the top of the jar was from the bottom of the meter stick for any particular *x*-value. The starting point, the meaning of 0, was arbitrary. It was used at the first frame in the movie for which the object was rolling freely. According to the model, at time 0 the object should have been 38.92 centimeters from the bottom of the meter stick. Compare this with the data point (0, 38) used to construct the model. The error, or residual, is the distance between the *y*-value obtained and the *y*-value predicted by the model. For this point, the error is $38 - 38.92$, or -.92 centimeters. In other words, the point on the scatterplot is .92 centimeters below the point on the graph.

Also notice the negative value for *a* in the regression model. This indicates that the parabola opens downward. This should become intuitive for students, as gravity constantly pulls downward on the object. Even though we roll the object up the plane, the force acting on the object increases its speed in a downward direction. The object will reach a maximum height, and then move downward.

B. Investigate the errors in the model.

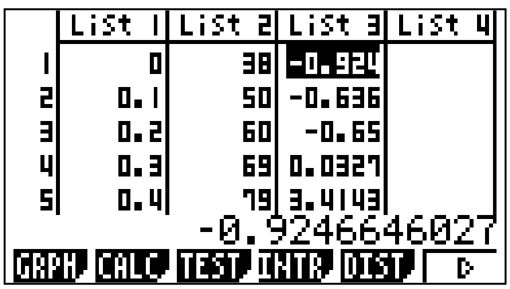
6) ***Calculate the residuals, and make a scatterplot of them to determine if the model is appropriate. Make conjectures as to underlying reasons for the residuals.***

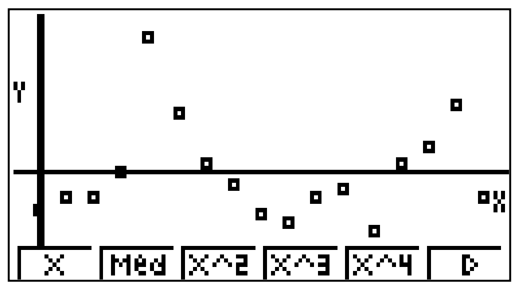
The errors, or residuals, in the model are the differences between the observed *y*-values (the position on the meter stick) and the *y*-values predicted by the regression equation. Because of what we did earlier in the SET UP for the Statistics, the residuals were automatically calculated and stored for us in List 3. The screen shown below left shows the beginning of this residual list.

A scatterplot of these residuals is often valuable. Unlike much in the world of mathematics, our hope is that there is no pattern to the residuals! If there is one, then our model is not the most appropriate for the data. Below right shows the scatterplot of residuals, using List 1 as the *x*-value and List 3 as the *y*-value. To obtain the graph, starting at the STAT menu:

- Press F1 for the GRPH menu.
- Press F6 to SET the graph.
- Press F2 to work on GPH2, making the appropriate choices for the scatterplot. Press EXIT when finished.
- Finally, press F2 to see the graph.

The residual plot, while perhaps not as chaotic as we might wish, does not show a clearly discernible pattern for our data points.

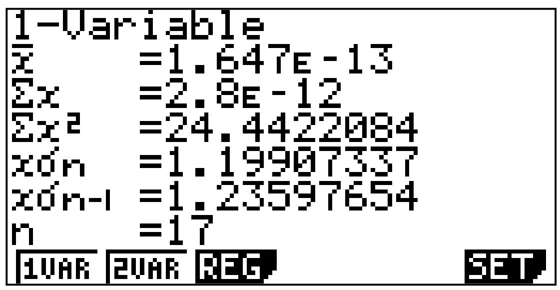




Many causes for errors in our regression model are likely. First and foremost, reading the measures from the camera required interpolation and some guesswork. Also, the object may not have been perfectly cylindrical, causing disruptions in its journey. Similarly, inconsistencies in the density of the object may have caused it to wobble. The table itself may not have been smooth. The angle of the roll may not have been proper, causing the object to twist somewhat on its path. The camera may have been somewhat inconsistent in shooting consecutive frames. Considering all of these likely sources of error, our model is remarkably good! Nevertheless, students who are accustomed to thinking of mathematics as unambiguous and even perfect may have difficulty accepting the idea that a mathematical model can be flawed.

7) Calculate the mean of the residuals, the sum of the residuals, and the sum of the squared residuals.

- From the STAT menu, choose F2 for CALC.
- Press F6 to SET the calculations up and F3 so that one-variable statistics are performed on List 3.
- Then press EXIT to back up to the "CALC" screen, followed by F1 to perform the one-variable calculations. The screen below shows the results.



Note that the mean of the residuals, denoted by $\bar{x}$, is extremely close to 0 as is the sum of the residuals, denoted by $\sum x$. Students new to regression may need some explanation as to why they should interpret the values shown on the calculator as 0. This is an important idea when using technology; calculators must use a finite number of significant digits when making calculations, so some rounding error often occurs. Students should also recognize that the average deviation and sum of deviations would be 0 for several models (e.g., a least-squares linear regression model, even a model that uses the average value of *y* as a predictor no matter the value of *x*). Consequently, using the sum of residuals to determine how well a model fits a particular data set is not very helpful. To avoid this, which is caused by the cancellation of positive errors by negative errors, the sum of squared errors is often used to compare mathematical models.

C. Explore the differences in successive distance measurements. Make a conjecture as to what these differences represent, and model them mathematically.

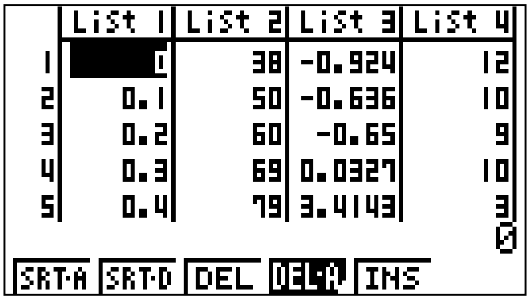
8) Create and investigate a list showing consecutive differences in the distances.

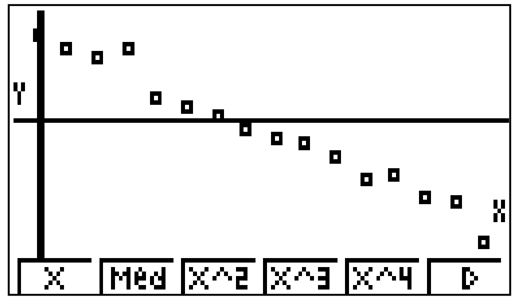
We will now create a list of successive differences to study, storing it in List 4. To do so:

- Move the cursor to the top of List 4.
- Press [OPTN] , [F1] for LIST, [F6] for more, and [F6] for more again.
- Now choose [F5] for ΔList and 2 for List 2. Then press [EXE] to perform the function. See the screen below left.

We are now interested in looking at a scatterplot of these values, using List 1 for the *x*-values and our new List 4 for the *y*-values. To set up the scatterplot:

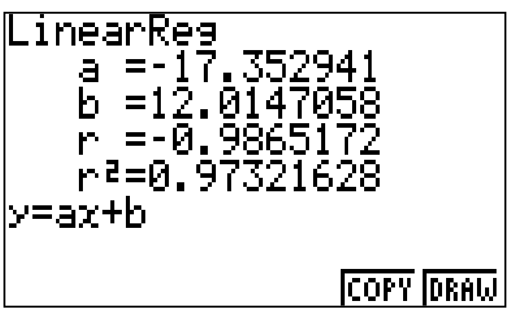
- Press [F1] for GRPH.
- Press [F6] for SET and [F3] for StatGraph 3. Then make sure you set up a SCATTER graph, using List 1 as the Xlist and List 4 for the Ylist. Press [EXIT] to back up a menu.
- Before you can display the graph, one more thing must be done. Since we are looking at differences, there is one less value in List 4 than there is in List 1. We need to delete the last value from List 1. To do so, highlight the last value, press [EXIT] to return to the primary “Statistics” screen and [F6] for more options. Scroll down to the bottom of List 1, and press [F3] to delete the last value.
- Then press [F6] for more options, [F1] for GRPH, and [F3] for GPH3. The results are shown below right.

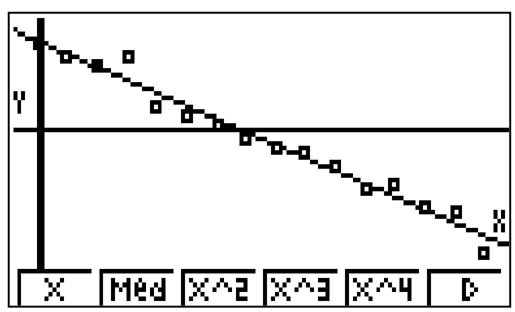




9) Interpret these differences and explore an appropriate regression model.

The scatterplot shows a reasonably consistent downward trend. Due to the consistency in the trend, a linear model may be most appropriate. In interpreting this, we should make use of the labels attached to the variables. The *y*-values show the distances traveled over successive time periods of .1 seconds. In other words, our y-values now measure velocity in centimeters per second. Positives reflect upward movement, and negatives reflect downward movement. The velocities should decrease, because gravity consistently pulls downward on the object. What we are actually seeing is that the velocity, the change in distance over time, is consistently decreasing. If we perform linear regression by pressing F1 on the scatterplot, we see the screen below left. The graph of this, obtained by pressing F6 for DRAW, is shown below right.





Note the high correlation of this linear model, indicating a good fit for the data. The *y*-intercept of 12.01 indicates that initially the jar's distance was changing at 12.01 centimeters per second. The positive value indicates that the object was moving upward. The slope, -17.35, shows that, on average, for every second, the velocity was decreasing at a rate of –17.35 centimeters per second.

D. Finally explore the second order differences. Make a conjecture as to what these differences mean, and model them mathematically.

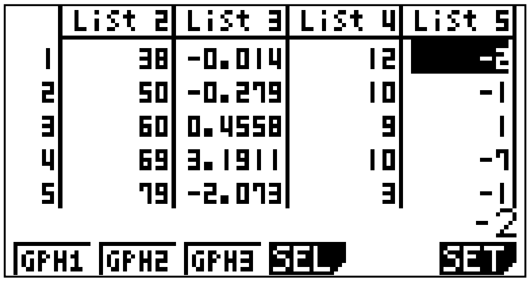
10) Create and investigate a list showing consecutive differences in the velocities.

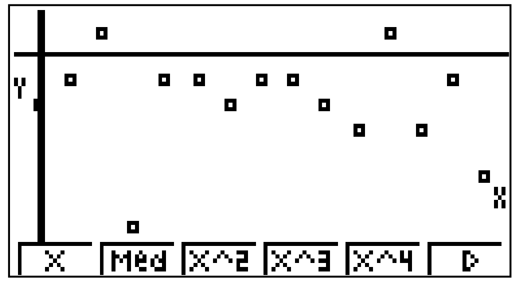
We will now investigate how much these velocities are changing. Just as we created a list showing the differences in the original distances (List 2), we will now create a new list, List 5, which will show the changes in velocity over time. The paragraphs below repeat the procedures we completed above, except that now we are investigating the differences in List 4. We will again use GRPH 3 for our scatterplot.

- Move the cursor to the top of List 5.
- Press [OPTN], [F1] for LIST, [F6] for more, and [F6] for more again.
- Press [F5] for ΔList and 4 for List 4. Press [EXE] to perform the function. See the screen below left.

Again, we wish to look at a scatterplot of these values, using List 1 as the *x*-value and our new List 5 as the *y*-values. To set up the scatterplot:

- Press [F1] for GRPH, [F6] for SET, and [F3] for StatGraph3. Then make sure you set up a SCATTER graph, using List 1 as the Xlist and List 5 for the Ylist. Press [EXIT] to back up a menu.
- Again, before you can display the graph, one more thing must be done. Since we are looking at differences, there is one less value in List 5 than there is in List 1. We need to delete the last value from List 1. To do so, highlight the last value, press [F6] for more options, and press [F3] to delete the last value.
- To return to the graph, press [F6] for more options, [F1] for GRPH, and [F3] for GPH3. The results are shown below right.



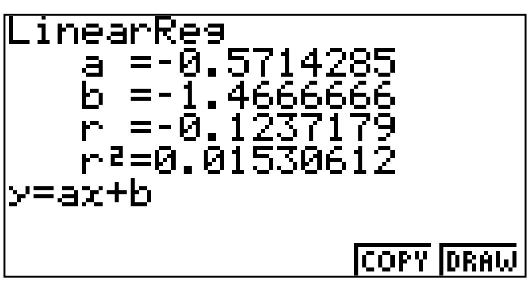


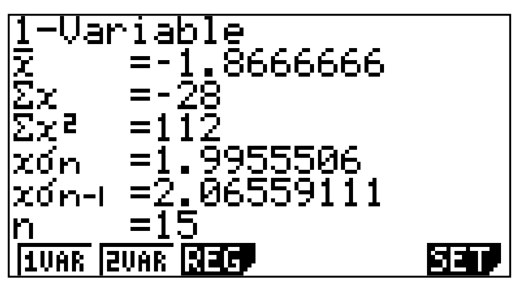
The graph seems to hang consistently a little below the *x*-axis. In other words, the velocity is changing at a negative rate, something that should not be very surprising since gravity was pulling downward on the object throughout its roll.

11) ***Explore and interpret an appropriate model for these differences***.

This time, linear regression (obtained by pressing [F1] on the scatterplot) produces a very low correlation. (See below left.) In other words, the change in velocity is not dependent on the velocity it is rolling at any particular time. If we perform one variable statistics on List 5, the calculator will determine the average change in velocity for us. To do so, from the screen below left,

- Press [EXIT] twice to back up to the primary "Statistics" screen and [F2] for "CALCULATIONS."
- Press [F6] for SET and [F5] so statistics are calculated on List 5.
- Press [EXIT] . Press [F1] for one-variable statistics. See below right.





The mean value of –1.867 tells us that, on average, each second the velocity was slowing down at the rate of 1.86 centimeters per second, or, as students may have heard, 1.86 centimeters per second squared.

An important point for students to explore here is that, although their scatterplots resemble lines, constant functions are not linear functions. This is because, for constant functions, the value of y does not depend on the value of x. Instead, the value of y remains the same (or, in statistics, approximately the same) no matter what x is.

The method described here was chosen for several reasons. One purpose of exploring the first and second order differences is to lay the underpinnings of the first and second derivatives many students will meet in calculus. Another purpose is for students to discover that the differences of second-degree functions are first degree, and that the differences of first-degree functions are constant (degree zero). Still a third purpose is for students to make some initial attempts at connecting mathematics and physics.

PROBLEM 2: STOPPING YOUR CAR

For any particular car, the distance it takes to stop is a function of the speed it is traveling when the brakes are applied. The following chart shows the distance it took a particular car to stop for the given speed.

SPEED (mph)	10	20	30	40	50	60	70
STOPPING DISTANCE (ft)	22	46	76	120	178	255	350

A. Construct a scatterplot.

B. Determine an appropriate model. Justify your selection.

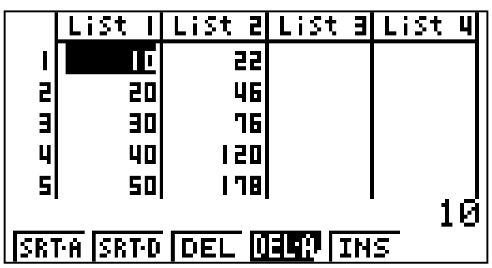
C. Using your scatterplot and model, write a short paragraph discussing your results.

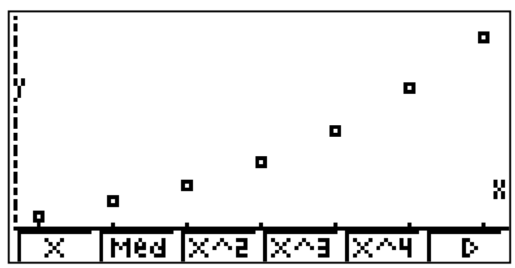
ONE SOLUTION TO PROBLEM 2: STOPPING YOUR CAR

A. Construct a scatterplot.

We must first enter the data into the calculator.

- From the LIST menu, clear two lists for the data. Pressing [F4] followed by [F1] will accomplish this.
- Type the data into the two lists. The beginning of the lists is shown below.
- Press [MENU] for the MAIN MENU and choose "Statistics."
- Press [F1] for GRAPH and [F6] to set it up. Make sure it is a scatter graph using the correct lists. Also, you may wish to check the SET UP. To do so, press [SHIFT] [MENU] from the primary "Statistics" screen. Make sure the window is set on AUTO. When you're ready, press [F1] twice from the primary "Statistics" screen to display the graph, assuming you set your graph up as Graph 1. Results are shown below right.



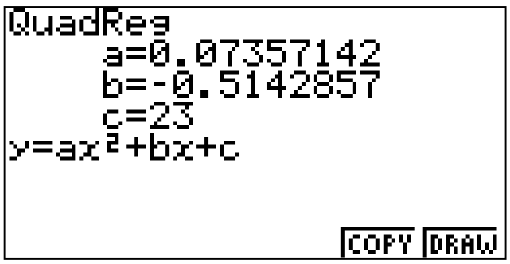


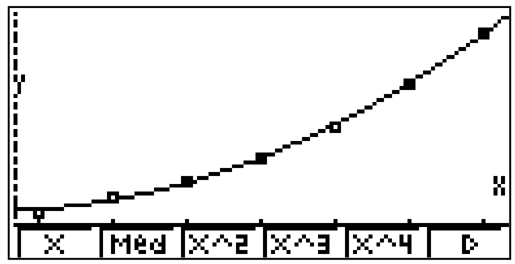
B. Determine an appropriate model. Justify your selection.

The model does not appear to be linear, and, in fact, is not. A quadratic is appropriate here. Consider how much distance is needed to slow down 10 miles per hour. Slowing down from 10mph to 0mph may not take too much. According to our data, it takes 22 feet. Slowing down from 20mph to 10mph should take a greater distance because we are traveling faster throughout the time interval. Another words, there cannot be a constant rate of change for the stopping distance. Slowing down from 60 to 50mph should take more than slowing down from 10 to 0mph. The model that is appropriate is quadratic.

To perform quadratic regression, while looking at the scatterplot:

- Press F3 for X^2 (quadratic) regression.
- Press F5 and EXE to copy the regression equation to Y1. See results below left.
- Press F6 to see the scatterplot and the regression function together. See results below right.





The regression equation is $y = .07357x^2 - .5143x + 23$.

C. Using your scatterplot and model, write a short paragraph discussing your results.

The regression curve does seem to fit the data extremely well. As our speed increases, our stopping distance increases, but at a faster rate. The positive value for *a* indicates that the parabola opens upward, which is consistent with our common sense understanding of the rate increasing as we move to the right on the graph.

The *y*-intercept of 23 feet (0, 23) is somewhat problematic. In theory, this says that if we are traveling at 0mph, it will take us 23 feet to stop! This, of course, is ludicrous. Ideally we would like the graph to go through (0, 0). We cannot just think of this as reaction distance either, because, assuming it always takes us a certain time to reach the brake, the distance we travel will be different when we travel at different speeds. Quite simply, we must recognize our model for what it is: an imperfect mathematical description of a real event. Nevertheless, it is still a very valuable method of making sense of our information.

PROBLEM 3: A FALLING OBJECT

An object is released and falls to the ground. Your goal is to investigate the object's fall as a function of the time it has been falling. Use a digital camera or motion detector and data collector.

A. Determine and calculate an appropriate mathematical model that relates how far the object has fallen with the length of time it has been falling.

B. Investigate the errors in the model, and make conjectures about their cause.

C. Explore the differences in successive height measurements, and extend this investigation into looking at the differences of these differences. Discuss your findings.

EXTENSIONS

1. What would happen if you measure the height of the object from the ground, rather than the distance it has fallen?
2. Investigate the changes that occur if you use different objects.
3. Investigate any changes that happen if you gather your data outside instead of inside.

PROBLEM 4: DISCOVERING A FORMULA

You may or may not recall the formula for the number of diagonals in a polygon. To see if you can find it on your own, draw several polygons, complete the chart below, and see if you can derive it by using a quadratic model.

# of Sides	3	4	5	6
# of Diagonals	0			

"BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

The ALGEBRA menu on the FX2.0 can be used to solve quadratic equations in a single variable.

- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation you desire and press [EXE] . For example, suppose you wish to solve the equation $x^2 + 5x = 9$. Simply display on the top line the following: solve($x^2 + 5x = 9$). When you press [EXE] , the calculator displays $x = -\frac{\sqrt{61}}{2} - \frac{5}{2}$ and $x = \frac{\sqrt{61}}{2} - \frac{5}{2}$. The "Algebra" menu can also be used to help students solve literal equations for y or any other variable.
- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press [EXE] . The top of the screen might look like this: solve ($2x^2 + x = 3 + 4y, y$). The calculator returns $y = \frac{x^2}{2} + \frac{x}{4} - \frac{3}{4}$.

Using the Algebra Menu to Solve Manually

In addition to solving equations, the FX 2.0 can also assist students in learning the process for themselves. From the MAIN MENU, access the "Algebra" menu and follow the steps below. This shows an example with two variables, but equations in one variable can also be solved using the same technique.

- To clear other entries, press the function key for "Clear," the number for "ALL EQUATIONS," and [EXE] for yes.
- At the cursor, type in the equation you want to work on and press [EXE] . For example you might type $2x^2 + x = 3$. This equation is labeled as equation 1.
- Suppose we wanted to solve this equation for x. All we need do to begin is simply tell the calculator to subtract 3. The calculator assumes you are referring to the equation in its memory. Type in "minus 3" in symbols and press [EXE] . This becomes equation 2, and should appear as $2x^2 + x - 3 = 3 - 3$.
- To simplify equation 2, press [F1] for the TRNS menu, the number for simplify, and [EXE] . The calculator displays $2x \ \ + x - 3 = 0$. This is labeled as equation 3.
- The next step could be to factor the quadratic expression. Simply press [F1] for the TRNS menu, the appropriate number for factor, the [F4] [3] [EXE] . The calculator shows $(2x + 3)(x - 1) = 0$.
- This equation needs to be solved, so again press [F1] for the TRNS menu, the number for solve [F4] [4] and [EXE] . The calculator shows that $x = 1$ and $x = -\frac{3}{2}$ using true fraction notation.

One of the many exciting features of the calculator is that it does whatever it is told to do, even if it does not help solve the problem. If the student tells the calculator to, say, multiply by 6 instead of divide by 6, the calculator does it, but the student can recognize that it does not help. The student can then return to the previous step and try something else.

Using the Tutorial Menu to Learn to Solve Linear Equations

The FX2.0 has a tutorial system built in that can help students to learn how to solve Linear Equations, Linear Inequalities, Quadratic Equations, and Simultaneous Equations. Within these, users can use equations already stored in the calculator or can input values for specific equations they wish to solve. The following demonstrates how the FX2.0 can help students solve the equation $3x^2 - 4 = x + 2$.

- From the MAIN MENU, highlight "Tutor" and press [EXE].
- Use the down arrow to highlight Quadratic Equation and press [EXE].
- Use the down arrow to highlight $Ax^2 + Bx + C = Dx^2 + Ex + F$ and press [F2] for "Input."
- Type in 3, 0 -4, 0, 1, 2 for A, B, C, D, E and F, respectively, pressing [EXE] after each.
- Press [F6] twice and the calculator will set up an automatic, step-by-step solution.
- Use [F6] and [F1] to move forward and backward through the solution. At each step of the way, the calculator indicates what it should do.

These techniques can be very effective tools in helping students master the skills they need to solve equations and inequalities. By "busting this barrier" that impedes the progress of so many students, the ALGEBRA FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	9.1, 9.2
AWSM – Focus on Advanced Algebra (1998)	5.1
Glencoe – Algebra 1 (1998)	1.2, 5.6, 11.1, 11.2
Glencoe – Algebra 2 (1998)	6.1, 6.2, 6.6
Holt Rinehart Winston – Algebra (1997)	2.1, 2.2, 12.1, 12.2, 12.4, 12.5
Holt Rinehart Winston – Advanced Algebra (1997)	5.1, 5.2, 5.3, 5.8
Key Curriculum – Advanced Algebra Through Data Exploration	
Merrill – Algebra 1 (1995)	13.1
Merrill – Algebra 2 (1995)	8.1, 8.3, 8.4, 8.5
McDougal Littell – Algebra 1: Explorations and Applications (1998)	8.2, 8.4
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	9.3, 12.4
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	8.8
Prentice Hall – Algebra (1998)	7.1, 7.2, 7.3, 7.5
Prentice Hall – Advanced Algebra (1998)	5.1, 5.5
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	9.1, 9.2, 9.3, 9.4
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	2.5, 6.6
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	10.1, 10.2, 10.3, 10.7