CLEMSON ALGEBRA PROJECT
UNIT 13: TRANSFORMATIONS

PROBLEM 1: FREE-FALL TRANSFORMATIONS

Suppose several objects are dropped at two-second intervals from a height of 1,450 feet, the height of the top of the roof of the Sears Tower in Chicago.

- Let x represent the number of seconds elapsed since the first object is dropped.
- Let B represent the number of seconds from the start when a particular object is dropped. For successive objects, B will be 0, 2, 4, 6, and so on.
- Let y represent the height of an object at any moment in time.
- Then the height of any of the objects is given by the formula

  $y = -16(x - B)^2 + 1450$.

In the formula, –16 is the result of the pull of gravity, which causes the objects to accelerate in a downward direction at 32 feet per second each second, and 1450 shows the initial height, the height from which the objects are dropped.

A. Explore the effects that the different values of B have on the graph. Interpret the changes in terms of the real world.

B. Focus on the first object, the one for which B is 0. First determine and explain the meanings of the x and y intercepts. Then, explore the effects of changing the maximum height from 1,450 to other values in increments of 50 feet.

C. Again focus on the first object. This time explore the effects on the graph if the gravitational constant is changed from –16. Consider the changes that would occur in places where gravity would be both more and less powerful.

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

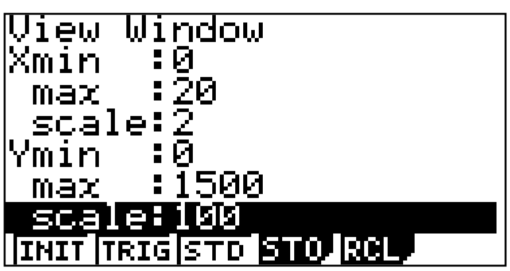
Explore the effects of changes of A, B, and C on equations in the form $y = Ax^2 + Bx + C$.

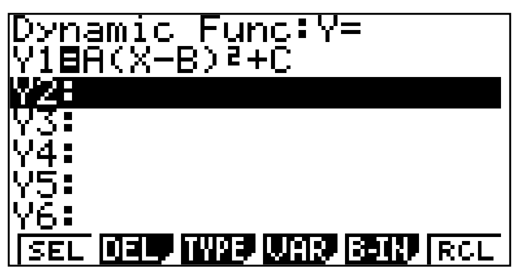
ONE SOLUTION TO PROBLEM 1: FREE-FALL TRANSFORMATIONS

A. Explore the effects that the different values of *B* have on the graph. Interpret the changes in terms of the real world.

This solution uses the dynamic graphing features of the calculators. To be in position to make changes to all of the parameters, we will use the form $y = A(x - B)^2 + C$. (NOTE: For students new to this, you may find it easier to use the form $y = -16(x - B)^2 + 1450$. The form used here was selected for greater flexibility.)

- From the MAIN MENU, choose "Dynamic."
- Either de-select any functions that are there with [F1] or delete them with [F2] followed by [F1].
- We'll first set the window. Press [SHIFT] [F3] to access the window. Recall that *x* represents the number of seconds since the first object is dropped, and *y* represents the height of the object, which is released from 1450 feet. A possible window is shown below left. To return to the primary "Dynamic" screen, press [EXIT] or [EXE].
- If necessary, press [F3] for type and select [F1] for Y =. Type in the right side of the function and press [EXE]. Your screen should look like the one below right.



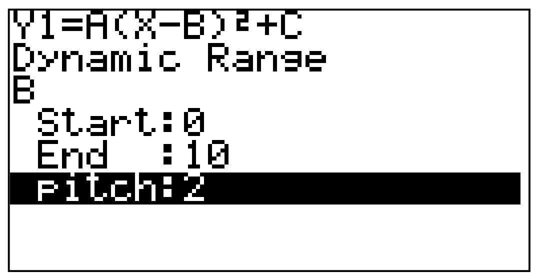


We now need to tell the calculator values for the parameters we want fixed and a range of values for the variable we want to be dynamic.

- Choose F4 for variables.
- Type in –16 for A. Press EXE .
- Press F1 to select *B* as the dynamic variable. Type in 0 for it. Press EXE .
- Type in 1450 for C and press EXE . See below left.

With these values set, we are now ready to set the RANGE for our dynamic variable.

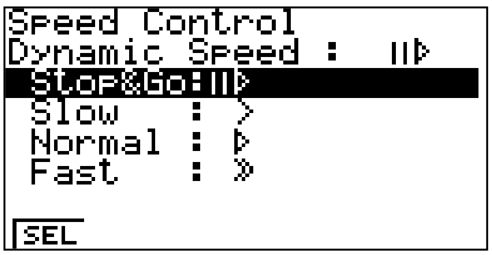
- Press F2 for RANGE. Type in the appropriate values, pressing EXE after each. See below right.

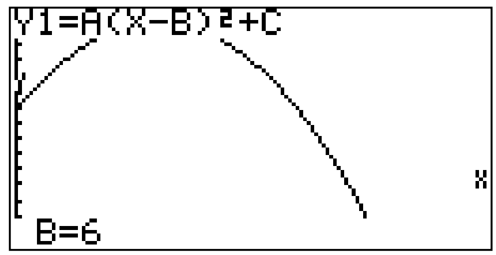


We also wish to control the speed of the graph. Pressing EXE after the RANGE has been set returns you to the VARIABLES screen shown above left. From this screen,

- Press F3 for SPEED.
- Select the option you want by highlighting it and pressing F1 . The “Stop&Go” option allows you to move the graph from one value to another by pressing EXE . This option is what has been chosen here. See below left.

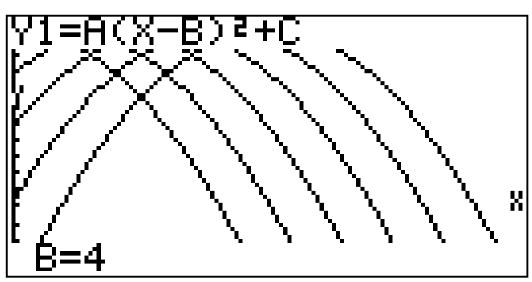
x After the Speed Control has been set, pressing [EXE] returns you to the screen shown above left. To begin viewing the dynamic graph, press [F6] for DYNAMIC. This will take a few seconds – your calculator is doing quite a bit of work! Pressing [EXE] from the screen will adjust the values of *B* according to the RANGE you have set. One such screen is shown below right.





Note a couple of features about the graph. First of all, you may wish to reset the window so that the maximum *y*-value is significantly higher. Also, since in the scenario the objects were being dropped, to interpret the graphs, we must recognize that the increasing component of the graph is not relevant here.

When you wish to stop looking at the graph, press [AC/ON] . The calculator also gives you the capability of viewing the graphs with the different values of *B* simultaneously. From the primary screen in the dynamic graphing mode, press [SHIFT] [MENU] for SET UP. Then set the LOCUS to ON. With this, the dynamic graph screen looks like the one below. Although it is not apparent from the picture, as you press [EXE] to move through the different values for B, the calculator highlights (in color on the 9850Ga Plus) the particular graph being displayed.

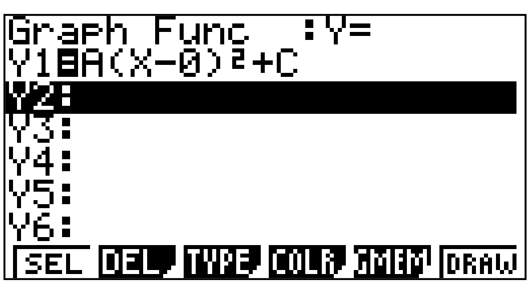


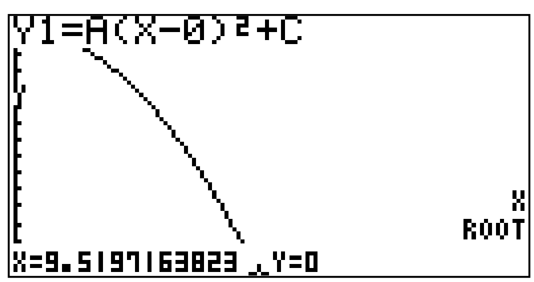
The conclusion that students should draw from this part of the exploration is that as *B* increases, the graph shifts to the right. In more formal terms, the graph of $f(x-B)$ translates the graph of $f(x)$ to the right *B* units. In terms of the real world, if *x* represents time, then so too must *B*. Furthermore, whatever relationship exists between *x* and *y* remains the same except that it happens *B* units of time (in our case seconds) later. That is, as *B* is increased by 2 seconds for our function, the graph shifts 2 units (seconds) to the right. As *B* is decreased by 2 seconds, the graph shifts 2 units (seconds) to the left.

B. Focus on the first object, the one for which *B* is 0. First determine and explain the meanings of the *x* and *y* intercepts. Then, explore the effects of changing the maximum height from 1,450 to other values in increments of 50 feet.

To explore the *x* and *y* intercepts, we will use the GRAPH menu.

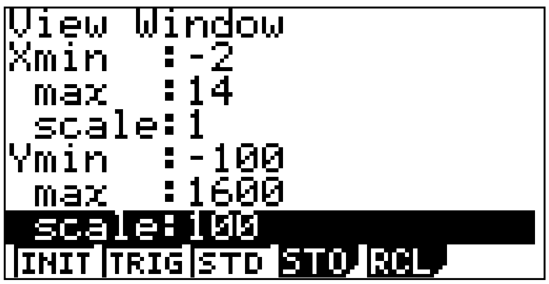
- From the MAIN MENU, choose the "Graph." Overtype *B* with 0 on Y1. See below left.
- Press [EXE] or [F6] to draw the graph.
- Press [F5] to access the Graph Solver.
- Press [F1] to find the root (*x*-intercept) of the function. This point, which has coordinates (9.52, 0), tells us that the object will hit the ground 9.52 seconds after it is released. See below right.
- Press [F5] to access the Graph Solver again. This time, press [F4] for the *y*-intercept. This point, (0, 1450), tells us that at time 0, the instant the object was released, the object was 1450 feet above the ground.





To explore the effects upon the graph of changing the maximum height, we could once again use the dynamic graphing features of the calculator. This time, however, we will take a different approach. Before doing so, we will change the window a little so that we can see higher values, have less blank screen on the right side of the graph, and can clearly see the axes.

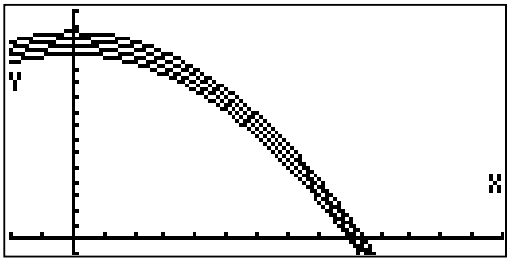
- Press SHIFT F3 to access the window, and type in appropriate values. See below left for one possible set of values. Press EXE when finished.
- This time, overtype *C* in the function with a range of values set in braces. The function now reads $Y = A(X - 0)^2 + \{1300, 1350, 1400, 1450\}$. Part of the screen is shown below right. Recall that *A* is still –16.

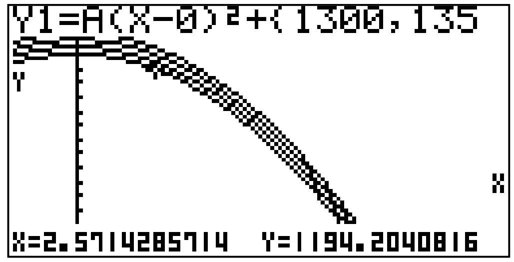


- From the screen above right, press F6 to draw the graph. The result is shown below left.

The change in values of *C* in increments of 50 feet shifts the graph vertically 50 feet. Because the graphs are not linear, the graphs may appear closer together as *x* increases; this is because as time increases the graphs that started higher are now "moving" faster. However, the vertical differences between the graphs remain 50 feet. To confirm this,

- Press F1 to trace.
- Use the right (and after you start, the left) arrow key to move to any point on the graph. Note the *y*-value of the point. See below right.
- Use the up and down arrow keys to move among the curves, maintaining the same *x*-value. Note how the *y*-values change in increments of 50.

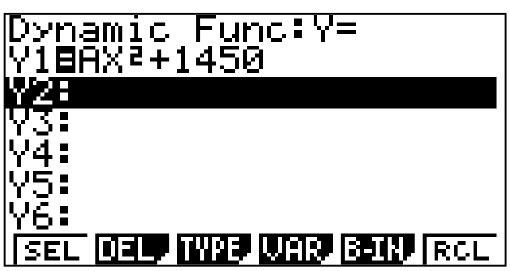


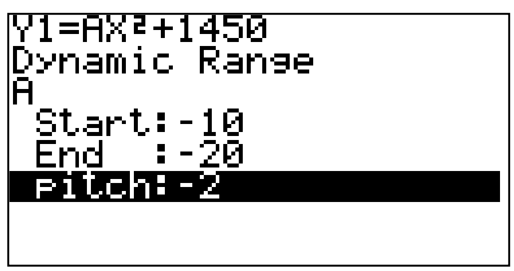


C. Again focus on the first object. This time explore the effects on the graph if the gravitational constant is changed from –16. Consider the changes that would occur in places where gravity would be both more and less powerful

The function we are now interested in investigating is $y = Ax^2 + 1450$, since *B* is 0 and C is 1450. This time we want *A* to vary. This could be done several ways, including using the dynamic graphing features of the calculator or setting different values for *A* in braces, as done in part *B* above. The dynamic feature is the method shown here.

- From the MAIN MENU, choose "Dynamic."
- Press [SHIFT] [MENU] for SET UP, and turn the Locus Off, if necessary.
- Type the function in for Y1 as shown below left.
- Choose [F4] for VARIABLE and then [F2] for RANGE. (Since *A* is the only parameter in the equation, it is automatically selected as the dynamic variable.)
- Select a desired range for *A*. One possible set of values is shown below right.

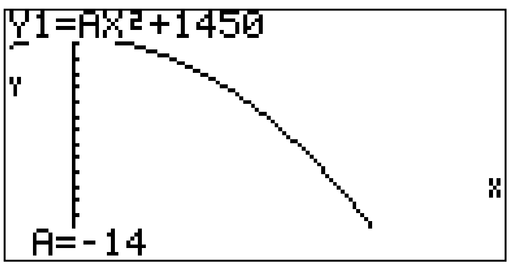


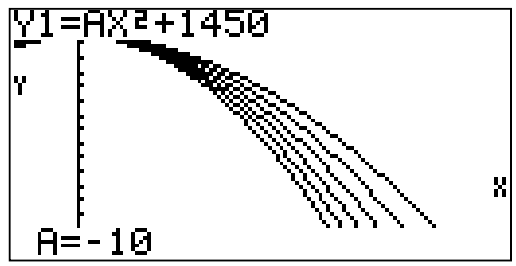


- Press EXIT after you have entered the values and F6 for the DYNAMIC graph. Again, be patient; your calculator is doing quite a bit of work!
- With the graph displayed, press EXE to move to the different values for *A*.

One screen is shown below left. If your window is different, your screen will look different. If desired, all of the graphs can be viewed simultaneously. From the graph:

- Press EXIT three times to return to the primary “Dynamic” screen.
- Press SHIFT MENU for SET UP.
- Turn the Locus on.
- Press EXIT and F6 to redisplay the graphs. See below right.





As the absolute value of *A* increases, the object picks up speed more quickly, causing the graph to turn down more quickly. Visually, the greater the absolute value of *A*, the “skinnier” the graph. Conversely, the smaller the absolute value of *A*, the “fatter” the graph. In context, as the effect of gravity becomes stronger, the absolute value of A increases and the curve becomes steeper more quickly.

PROBLEM 2: ABSOLUTE VALUE MOTION

One general form of the absolute value function is $y = A|x - B| + C$. This function can be explored with the MOTION program on your calculator. For this program, *x* represents the number of seconds since you pressed the trigger on the data collector, and *y* represents your distance from the motion detector. The purpose of this investigation is to use this program to discover the effects that *A*, *B*, and *C* have on the absolute value function.

A. Explore the effects that changes in *C* have on the vertex of the graph.

Type in 5 for *B*.

Type in 0.5 for *A*.

Then explore the function with *C* being first 1, then 2, and finally 3.

B. Explore the effects that changes in *B* have on the vertex of the graph.

First type in 3, then 4, and finally 5 for *B*. Each time, do the following:

Type in 0.5 for *A*.

Type in 2 for *C*.

C. Explore the effects that changes in *A* have on the graph.

Type in 5 for *B*.

Try *A* equal first to -0.4, then .1, and finally 0.6.

Type in 3 for *C*.

D. Summarize the effects *A*, *B*, and *C* have on the graph

ONE SOLUTION TO PROBLEM 2: ABSOLUTE VALUE MOTION

In this problem, we have shifted from a quadratic function to an absolute value function in the form $y = A|x - B| + C$. The goal of the problem is to help students recognize that *A*, *B*, and *C* have identical effects on this function as they do on the quadratic function.

We assume that students are familiar with the MOTION program, which is described in the unit on Linear Functions. Again for this activity, the class should break into groups of size 3 or 4. Each person should have a responsibility, and these responsibilities should rotate through the group as the problem is explored. As suggested in the unit on Linear Functions, for a group with four people, one person can be the "techie," assigned to make sure the equipment is hooked up properly and the path for the walker is clear. A second person can be the "trigger person," giving the commands to the data collector and the calculator. The third person can be the "commander," telling the "walker" where and when to start, as well as the direction and speed to go; and the fourth person can be the "walker."

Once the duties for each person have been established,

- Clear a path about 6 meters long (for walking) and about 2 meters wide.
- Link the calculator to the data collector.
- Turn the data collector on.
- Connect the motion detector to the data collector's sonic port. Set the motion detector at the end of the walking path at a height that will "hit" the walker.
- Turn on the calculator and select "Program" from the MAIN MENU.
- Call up the MOTION program on the calculator.
- Select option 4, WALK MY EQUATION and press EXE .
- Then choose option 2, ABSOLUTE VALUE, and press EXE .
- Although not always stated below, continue to press EXE after you have made a selection in the program.

A. Explore the effects that changes in *C* have on the vertex of the graph.

- WALK MY EQUATION asks that you put in values for *A*, *B*, and *C*, but not in this order. Follow the instructions on the screen to enter the values for *B* (put in 5), *A* (put in .5) and *C* (use 1 the first time). In other words, you are to first try to "walk" the function $y = 0.5|x-5|+1$.
- After you have typed in the equation, the instructions on the calculator tell you to press [TRIGGER] on the EA-100 (the data collector) when you are ready to begin and [EXE] on the calculator when the data have been collected.

When you have completed this, you will see on the calculator the plot you have walked marked with Xs along with the graph that you were trying to walk. Discuss with your group how you could improve the walk.

- Press [EXE] when finished to return to the program. Again select ABSOLUTE VALUE. Either try the first part again, or move on to the second, choosing 2 for *C* instead of 1.

Again after finishing, either try again or move on to the third problem, choosing 3 for *C*. When they have completed this part of the exercise, students should note that different values of *C* shift the vertex of the graph vertically. Because it shows vertical displacement, it is measured in the same units as *y*, in this case meters. As *C* moves from 1 to 2 and from 2 to 3, the vertex of the graph shifts from (5, 1) to (5, 2) and then from (5, 2) to (5, 3). In terms of the real world, the turn-around point in all cases happens five seconds after starting and at distances of 1, 2, and 3 meters, respectively, from the motion detector.

B. Explore the effects that changes in *B* have on the vertex of the graph.

- The first time you attempt this, type in 3 for B. On the second and third trials, type in 4 and 5 respectively. Each time, do the following.
- Type in .5 for A.
- Type in 2 for C.

The first time you are trying to walk $y = 0.5|x - 3| + 2$. Again follow the directions on the calculator, which tell you to press [TRIGGER] on the data collector to begin collecting data and [EXE] on the calculator when finished. Compare your walk, marked with Xs, to the actual graph. Press [EXE] to continue. Either try again, or move on to the next part, inputting 4 for *B* and eventually 5 for *B*.

The goal is to note what happens to the vertex of the absolute function. When *B* is 3, the turn-around is at (3, 2). In other words, the direction changes 3 seconds after beginning and at a distance of 2 meters away from the motion detector. When *B* is 4, the turn-around is at (4, 2). In other words it is now 4 seconds after starting and again at a distance of 2 meters away from the motion detector. Finally, when *B* is 5, the turn-around is at (5, 2), five seconds after starting and still at a distance of 2 meters away from the motion detector.

Students may also note the changes in *y*-intercepts created by the changes in the values for B. This too is an excellent realization, but not one that is quite so easily interpreted. The *y*-intercepts of the three functions, using 3, 4, and 5 for *B*, are, respectively, at (0, 2), (0, 2.5), and (0, 3). These tell the students that their starting points differ according to what *B* is. They may also note that the speed and direction with which they are to walk does not change, but the length of time in which they walk in a particular direction is also affected.

C. **Explore the effects that changes in *A* have on the graph.**

- Type in 5 for *B*.
- Try *A* equal first to -0.4, then .1, and finally 0.6.
- Type in 3 for *C*.

Again using WALK MY EQUATION on the MOTION program, you are exploring the differences among the equations $y = -0.4|x-5|+3$, $y = 0.1|x-5|+3$, and $y = 0.6|x-5|+3$. After attempting each walk, press [EXE] to return to the program. Either try again or move on to the next part of the problem.

The first thing students will probably notice is that the negative value for *A* turns the absolute value function upside down. When *A* is positive, the graph looks like the "V" shape they may have grown to associate with the absolute value function.

Another item they should discover is that the vertex for all of the graphs is at (5, 3). This tells us that five seconds after they start they should be three meters away from the motion detector, and it is at that point that they should change directions.

The third important point for students to recognize is that *A* affects the slope of the graph, which tells them the speed at which they need to walk. They may discover this by first calculating the *y*-intercept, the starting point for their walk. This is an excellent realization, but one which the teacher may have to give help on so that they make the connection to the speed. Because speed is distance covered per time, and *y* represents distance measured in meters and *x* represents time measured in seconds, the slope of the graph represents change in distance over change in time. The teacher should lead students toward the connections among *A*, the slope of the graph, and the speed (as well as direction) at which they need to walk. For the three values of *A* used here, the walks need to be at 0.4 meters (40 centimeters) per second, starting toward the detector; at 0.1 meters (10 centimeters) per second starting away from the detector; and at 0.6 meters (60 centimeters), starting away from the detector, respectively.

D. Summarize the effects A, B, and C have on the graph of $y = A|x - B| + C$ **.**

From this work, students should discover that the effects that *A*, *B*, and *C* have on the absolute value function are analogous to their effects on the quadratic function. *A* affects the slope of the graph, which represents the speed and direction of the walk. As the absolute value of *A* increases, the graph becomes "skinnier." Positive values for *A* cause the graph to open upward, and negative values cause it to open downward.

B is measured in the same units as *x*, in this case seconds. As *B* increases, the graph shifts to the right, and as *B* decreases, the graph shifts to the left. For example, changing *B* from 0 to 2 shifts the graph 2 units (seconds) to the right.

C is measured in the same units as *y*, in this case meters (or feet, if the units have been changed on the OPTIONS component of the program). As *C* increases, the graph shifts upward, and as *C* decreases, the graph shifts downward. For example, changing *C* from 0 to 2 moves the graph 2 units up.

PROBLEM 3: TRANSFORMING MONEY GROWTH

Suppose you have some money to invest for a period of several years. Because interest is compounded (in the second year, for example, you get interest not only on the principal but on the interest you have already earned), the mathematical model that represents this is exponential. For this investigation, consider the general form for an exponential function, $y = a * b^x$. For our function:

- *a* represents the amount of the original investment.
- *b* represents 1 + the interest rate, the multiplier needed to determine the value of the investment in each successive year.
- *x* represents the number of years that the money has been invested.
- *y* represents the total amount the investment is worth at a given point in time.

Begin with the supposition that you invest $1000 at 4% annual interest for a period of 25 years. First graph this and determine the final value of your investment. Then, use the dynamic graphing features of the calculator (with *x* representing time) to investigate the effects of the following:

A. Vary the initial investment from $1000 to $10,000. What happens to the graph and to the final value of the investment?

B. Vary the interest rate from 4% up to 14% in increments of 2%. What happens to the graph and to the final value of the investment?

C. Suppose you decide to wait *c* years before investing your money. Investigate the effects on the graph and on the final value of the investment if c varies from 0 to 6 years.

Hint: The function becomes $y = a * b^{(x-c)}$.

PROBLEM 4: ELLIPSE PLACEMENT

Suppose you were drawing a picture on your graphing calculator and needed an ellipse as part of the figure. Using the SET UP under the "Conics" menu, turn the Grid, Axes, and Label Off. Then, set the window so that the domain is [-12, 12] and the range is [-8, 8]. Use a scale of 0 on both. After selecting the appropriate type of equation for the ellipse, draw an ellipse with $A = 4$, $B = 3$, $H = 0$, and $K = 0$. Assume you now need to adjust the ellipse for your picture.

A. You decide to move the center of the ellipse from the middle of the screen to the right half of the screen. What do you do?

B. You determine that the center should now be about 3/4 of the way between the top and the bottom of the screen. What do you do?

C. You now decide that the horizontal axis of the ellipse should be one-and-a-half times as long as you originally thought. What do you do?

D. You also determine that the vertical axis of the ellipse should be only two-thirds as long as you originally thought. What do you do?

If you like, you can store your ellipse and use it as part of a bigger picture. When it is drawn, press [OPTN] , [F1] for PICT, [F1] to store it, and the function key of the picture memory location you want to use. You can then choose this picture as the background from the SET UP menu.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	**SECTION**
AWSM – Focus on Algebra (1998)	
AWSM – Focus on Advanced Algebra (1998)	5.1
Glencoe – Algebra 1 (1998)	
Glencoe – Algebra 2 (1998)	7.4
Holt Rinehart Winston – Algebra (1997)	9.2, 9.3
Holt Rinehart Winston – Advanced Algebra (1997)	
Key Curriculum – Advanced Algebra Through Data Exploration	
Merrill – Algebra 1 (1995)	
Merrill – Algebra 2 (1995)	
McDougal Littell – Algebra 1: Explorations and Applications (1998)	
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	
Prentice Hall – Algebra (1998)	
Prentice Hall – Advanced Algebra (1998)	1.5, 10.4, 10.6
SFAW: UCSMP – Algebra Part 1 (1998)	3.4, 6.7
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	6.3
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	