CLEMSON ALGEBRA PROJECT
UNIT 14: CONIC SECTIONS

PROBLEM 1: LORAN - LONG-DISTANCE RADIO NAVIGATION

LORAN, long-distance radio navigation for aircraft and ships, uses synchronized pulses transmitted by widely separated transmitting stations. These pulses travel at the speed of light (186,000 miles per second). The difference in the times of arrival of these pulses at an aircraft or ship is constant on a hyperbola which has transmitting stations located at the foci. Assume that two stations, 300 miles apart, are positioned on a rectangular coordinate system at points (-150, 0) and (150, 0) and that a ship is traveling on a path with coordinates (x, 75).

A. Find the x-coordinate of the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).

B. Write the equation of the hyperbola on which the ship is located.

REFERENCE: *Precalculus, Third Edition,* by Larson and Hostetler, D.C. Heath and Company, 1993.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

ONE SOLUTION TO PROBLEM 1: LORAN

A. Find the *x*-coordinate of the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).

The difference in the distance from a point on the hyperbola to the foci is constant. In this instance, use the time difference in receipt of the pulses to determine that constant difference.

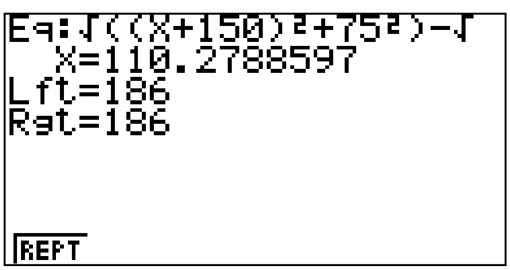
Time travels at 186,000 miles per second. Because the time difference is 0.001 second, the constant difference must be 0.001 seconds times 186,000 miles per second, or 186 miles.

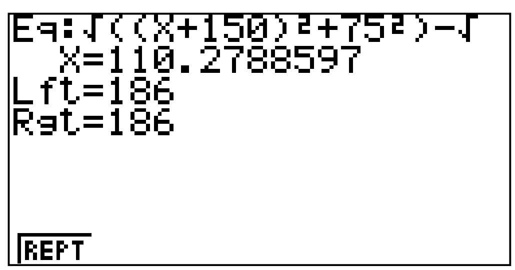
Next, we'll use the distance formula to find an expression that describes the distance from each of the foci to the location of the ship. The equation is:

$$\sqrt{(x+150)^2+75^2}-\sqrt{(x-150)^2+75^2}=186$$

To solve, this equation, we will use the "Equation" mode on the calculator. From the MAIN MENU, choose "Equation," and then:

- Press F3 for the "Solver."
- Type in the equation, being careful with parentheses – make sure you include parentheses around the entire expressions under the radicals. See below left for the beginning of the equation. (NOTE: Your calculator may show something different for *X* at this point.)
- Press F6 to solve the equation. See below right.





Our solution is $X = 110.28$. This tells us that the ship is located at the point which has coordinates (110.28, 75).

B. Write the equation of the hyperbola on which the ship is located.

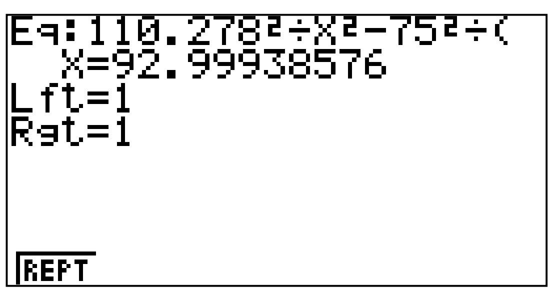
The general form for our hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The relationship between *a* (the distance from the center to a vertex), *b* (the distance from the center to the conjugate axis), and *c* (the distance from the center to a focus) is $a^2 + b^2 = c^2$. Consequently, $b^2 = c^2 - a^2$.

We already know that the foci are at (-150, 0) and (150, 0), giving us a value of 150 for *c*. Substituting, we have that $b^2 = 150^2 - a^2$.

Making another substitution, the equation for our hyperbola becomes $\frac{x^2}{a^2} - \frac{y^2}{150^2 - a^2} = 1$. We can now substitute the *x* and *y* values of the point we found in part A into this equation. This gives us an equation with *a* as the only variable, $\frac{110.278^2}{a^2} - \frac{75^2}{150^2 - a^2} = 1$.

Once again, we will use the equation solving capabilities of the calculator. From the MAIN MENU, call up “Equations.” Then,

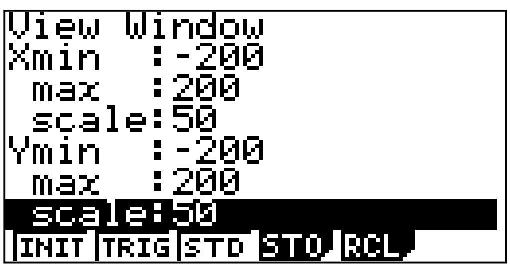
- Press [F3] for “Solver.”
- Type in the equation. Be careful with parentheses, and use *x* instead of *a*. Press [EXE] .
- Press [F6] to solve the equation. See below.

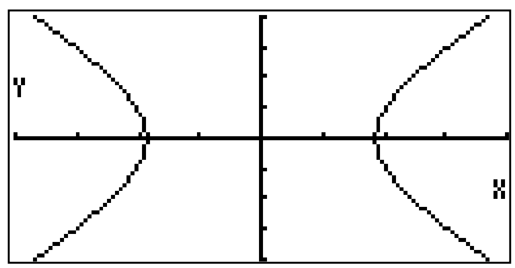


We now have a = 93.00 and c = 150. From the MAIN MENU, choose "Run," and find $\sqrt{150^2 - 93.00^2}$ to find b. You should get b = 117.69. Thus our equation is $\frac{x^2}{93.00^2} - \frac{y^2}{117.69^2} = 1$.

To graph this hyperbola, from the MAIN MENU, call up "Conics." Then,

- Set the viewing window by pressing [SHIFT] [F3] . Remember to press [EXE] after each entry. One possible window is shown below left. Press [EXIT] when finished.
- Then, use the down arrow to highlight the horizontal hyperbola and press [EXE] . Type in 93.00 for A, 117.69 for B, 0 for H, and 0 for K, pressing [EXE] after each entry.
- Press [F6] to draw the graph. See below right.





- You can trace points on the hyperbola by pressing [F1] and using the arrow keys.
- If you press [F5] you will access the "Graph Solver." Note you can find the foci, the intercepts, and the vertices with this.
- To draw asymptotes, press [F5] twice.

PROBLEM 2: GEOLOGY AND EARTHQUAKES

When an earthquake occurs, energy waves radiate in concentric circles from the epicenter, or the point above which the earthquake occurred. Stations with seismographs record the level of that energy and how long the energy took to reach the station.

A. Suppose one station determines that the epicenter of an earthquake is about 100 miles from the station. Find an equation for the possible location of the epicenter.

B. A second station, 120 miles east and 160 miles south of the first station, shows the epicenter to be about 135 miles away. Find an equation for the possible location of the epicenter.

C. Using the information from parts A and B, find the possible locations of the epicenter.

REFERENCE: *Advanced Algebra*, Holt, Rinehart and Winston, 1997.

ONE SOLUTION TO PROBLEM 2: GEOLOGY AND EARTHQUAKES

A. Suppose one station determines that the epicenter of an earthquake is about 100 miles from the station. Find an equation for the possible location of the epicenter.

Assume that the location of the station is at (0, 0) on a coordinate grid. The epicenter could be any place on a circle which has its center at (0, 0) and a radius of 100. Our equation is $x^2 + y^2 = 100^2$.

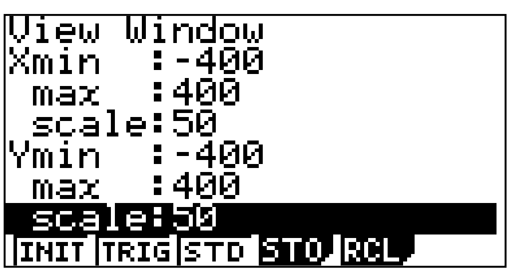
B. A second station, 120 miles east and 160 miles south of the first station, shows the epicenter to be about 135 miles away. Find an equation for the possible location of the epicenter.

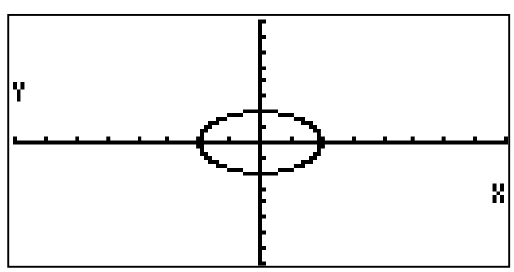
An equation to locate the epicenter would be a circle, with center (120, -160) and radius 135. Its equation is $(x - 120)^2 + (y + 160)^2 = 135^2$.

C. Using the information from parts A and B, find the possible locations of the epicenter.

We will use a graphing solution to find the possible locations of the epicenter. From the MAIN MENU, choose "Conics."

- Down arrow to highlight the circle and press [EXE] .
- Press [SHIFT] [F3] to set the viewing window. A reasonable window is shown below left. Remember to press [EXE] after entering each value. Press [EXIT] when finished.
- Type in 0 for H, 0 for K, and 100 for R to draw the first circle. Press [EXE] after each entry and then [F6] to draw the circle. See below right.





The circle appears as an ellipse because of the window we have chosen.

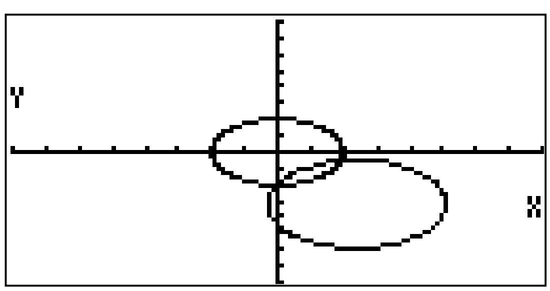
We want to have a second circle drawn. What we need to do is save the graph of this first circle as a picture that we will use as background. With circle showing,

- Press OPTN and then F1 for "Picture."
- Press F1 to store the picture and F1 again for picture memory 1. (Use a different picture memory location if desired.)

Our next step is to use this picture as background.

- Press SHIFT MENU for the SET UP.
- Move the down arrow to highlight Background.
- Press F2 for "Picture" and F1 for picture 1. Press EXIT .

- We're now ready to add our second circle. From the circle screen, type in 120 for H, -160 for K, and 135 for R, pressing EXE after each entry.
- Press F6 to draw the circle. The first circle should be there too. See below.
- Press F1 for "TRACE" and use the right and left arrow keys to move around the second circle. The points of intersection are located approximately at (95, -27) and (-2, -102).



PROBLEM 3: MARINE BIOLOGY

Hyperbolas can be used to locate objects underwater. To locate a whale in the ocean, two microphones are placed 8000 feet apart. One microphone picks up a whale noise 0.4 seconds after the second microphone picks up the same noise. The speed of sound in water is about 5000 feet per second.

A. How much farther from the whale is the first microphone?
B. Find an equation for the possible locations of the whale.
C. What is the closest distance that the whale could be to the second microphone?
D. Will the whale always be closer to the microphone that receives the signal first? Can the whale be on either branch of the hyperbola? Explain your reasoning.

REFERENCE: *Advanced Algebra,* Holt, Rinehart and Winston, 1997.

PROBLEM 4: MOUNTAIN TUNNEL

A semi-elliptical arch over a tunnel for a road through a mountain has a base at the opening of 100 feet. The height at the center of the tunnel is 30 feet. Determine the height of the arch 5 feet from the outside edge of the tunnel.

REFERENCE: *Precalculus, Third Edition,* by Larson and Hostetler, D.C. Heath and Company, 1993.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	
AWSM – Focus on Advanced Algebra (1998)	5.3
Glencoe – Algebra 1 (1998)	
Glencoe – Algebra 2 (1998)	7.3, 7.4, 7.5, 7.6
Holt Rinehart Winston – Algebra (1997)	
Holt Rinehart Winston – Advanced Algebra (1997)	10.2, 10.3, 10.4
Key Curriculum – Advanced Algebra Through Data Exploration	12.5, 12.7
Merrill – Algebra 1 (1995)	
Merrill – Algebra 2 (1995)	9.3, 9.4, 9.5, 9.7
McDougal Littell – Algebra 1: Explorations and Applications (1998)	
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	
Prentice Hall – Algebra (1998)	
Prentice Hall – Advanced Algebra (1998)	10.1, 10.3, 10.4, 10.5
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	12.2, 12.3, 12.4, 12.6, 12.7
Southwestern – Algebra 1: An Integrated Approach (1997)	