

CLEMSON ALGEBRA PROJECT UNIT 2: EQUATIONS IN ONE VARIABLE

PROBLEM 1: THE FIELD TRIP

The American History Club is taking a field trip to Philadelphia and Boston. The club has raised enough money to provide transportation and lodging for everyone. However, each student must take enough money to cover meals and extras. You plan to take \$425.00 and spend \$30.00 each day. Your friend is taking \$550.00 and is planning to spend \$45.00 each day.

- A. Write an algebraic expression for the amount of money you will have remaining at the end of any day. Define any variables you use.
- B. Write an algebraic expression for the amount of money your friend will have remaining at the end of any day. Define any variables you use.
- C. Use a table to determine the day on which you and your friend will have exactly the same amount of money remaining.
- D. Explain how you decided on which day you would both have the same amount of money.
- E. Write an equation to determine when you and your friend would have the same amount of money. Use the equation solver on your calculator to solve this equation. How does this answer compare with your answer from problem C? Explain any differences in the answers.
- F. Exactly how much money would you both have on this day?
- G. At most, how many days would you like the field trip to last? Do you think your friend would agree with you? Explain your reasoning.
- H. What is the maximum amount your friend could spend each day in order to be able to take a 21-day trip and only spend the \$550.00? Justify your reasoning.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

EQUATIONS IN ONE VARIABLE

ONE SOLUTION PROBLEM 1: THE FIELD TRIP

- A. Write an algebraic expression for the amount of money you will have remaining at the end of any day. Define any variables you use.**

We'll start by identifying a variable to help us. Let X = the number of the day on the trip. Since we start with \$425, after one day we'll have $425 - 30$ dollars. After two days we'll have $425 - 30 * 2$ dollars left. After 3 days, we'll have $425 - 30 * 3$ dollars left. After X days, we'll have $425 - 30X$ dollars left.

- B. Write an algebraic expression for the amount of money your friend will have remaining at the end of any day. Define any variables you use.**

Again we'll let X = the number of the day on the trip. Using logic similar to that in part A, after X days, our friend will have $550 - 45X$ dollars left.

- C. Use a table to determine the day on which you and your friend will have exactly the same amount of money remaining.**

From the MAIN MENU, call up the "Table" function. Then,

- x If necessary, press **F3** to choose the "Type," followed by **F1** for "Y=."
- x Type in $425 - 30X$ for Y1, using the **X, θ , T** key for X. Press **EXE**.
- x Type in $550 - 45X$ for Y2. Press **EXE**. See below left.
- x If the equal sign in Y1 or Y2 is not highlighted, move the cursor to the function and press **F1** to select it.
- x Next, we'll set the range of values for X. Press **F5** to access the "Range" function. Since X represents the number of days on the trip, we might want to start at day 0 and end by day 15 (we would already be out of money). The "pitch" refers to the increment the calculator will use for X. Press **EXE** after each value you enter. See below right. Press **EXIT** when finished.

EQUATIONS IN ONE VARIABLE

```

Table Func :Y=
Y1 425-30X
Y2 550-45X
Y3:
Y4:
Y5:
Y6:
SEL DEL TYPE CLR RANG TABL
    
```

```

Table Range
X
Start:0
End :15
Pitch:1
    
```

The table should have the values shown below. You can use the arrow keys to move through the table. From the data shown, we determine that sometime during the 9th day you will have the same amount of money as your friend.

X	Y1	Y2
1	395	505
2	365	460
3	335	415
4	305	370
5	275	325
6	245	280
7	215	235
8	185	190
9	155	145
10	125	100
11	95	55
12	62	10
13	35	-35
14	5	-80
15	-25	-125

EQUATIONS IN ONE VARIABLE

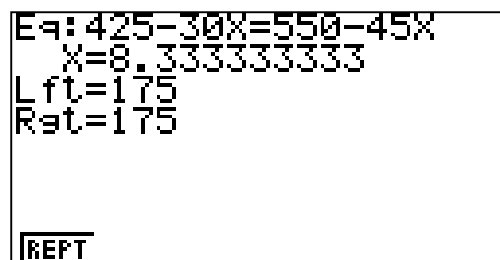
D. Explain how you decided on which day you would both have the same amount of money.

After examining the table of values, the students should recognize that after the eighth day, the values in the Y2 column become less than the values in the Y1 column. Therefore at some point during the ninth day, the two students will have the same amount of money. Students may estimate that both students have approximately \$170.00.

E. Write an equation to determine when you and your friend would have the same amount of money. Use the equation solver on your calculator to solve this equation. How does this answer compare with your answer from problem C? Explain any differences in the answers.

From the MAIN MENU, call up the “Equation” function. Then,

- x Press **F3** for the “Solver.”
- x Type in the equation, using **SHIFT** **.** for the equal sign. See below left.
(NOTE: At this point, you may have a different value for X from the one shown on the screen.)
- x Press **F6** to solve the equation. See below right.



Our result tells us that after $8\frac{1}{3}$ days (part way into the ninth day of the trip), you will have the same amount of money as your friend. Further, the calculator shows that, for this value for X , both sides of the equation evaluate to 175. Thus, after $8\frac{1}{3}$ days, we will both have \$175. This is consistent with our estimate from the table.

EQUATIONS IN ONE VARIABLE

F. Exactly how much money would you both have on this day?

We already answered this in looking at the screen for part E. Both the left and right sides of our equation evaluate to 175, indicating we both have \$175.

G. At most, how many days would you like the field trip to last? Do you think your friend would agree with you? Explain your reasoning.

The student may examine the table and determine that at the end of the 14th day \$5.00 remains, thereby concluding that the trip should last no longer than 14 days. She/he may also use the equation solver to determine how long the trip should last if all of the money is to be spent. We can determine that your money will be gone in a little over 14 days (see below left), but that your friend's money will be gone in a little over 12 days (see below right).

```
Eq: 425 - 30X = 0
  X = 14.16666667
Lft = 0
Rst = 0

[REPT]
```

```
Eq: 550 - 45X = 0
  X = 12.22222222
Lft = 0
Rst = 0

[REPT]
```

Therefore the friend would not want to take a trip that would last 14 days.

H. What is the maximum amount your friend could spend each day in order to be able to take a 21-day trip and only spend the \$550.00? Justify your reasoning.

One way to explore this problem is with a table, looking at various expressions. From the MAIN MENU, call up the "Table" function. Enter the following expressions, pressing **[EXE]** after each entry. (See below left.)

Y1: 550 - 45X

Y2: 550 - 40X

Y3: 550 - 35X

Y4: 550 - 30X

Y5: 550 - 25X

Y6: 550 - 20X

EQUATIONS IN ONE VARIABLE

- x Next, press **F5** to set the “Range.” You may wish to start at 0 and end at a value like 25, again using a pitch of 1. Press **EXE** after each value you type in (if the correct value is there, use the down arrow to move to the next field). See below right. When all values have been entered, press **EXIT** .

<pre> Table Func :Y= Y1 550-45X Y2 550-40X Y3 550-35X Y4 550-30X Y5 550-25X Y6 550-20X SEL DEL TYPE COLR RANG TABL </pre>	<pre> Table Range X Start:0 End :25 Pitch:1 </pre>
---	--

- x Press **F6** to see the table.

After examining the table of values created, the student should determine that if the friend spent \$25.00 per day, at the end of 22 days, the friend would have no money remaining. Therefore, a trip for 21 days would be possible. Since we used increments of \$5 in our functions, we may wish to see if the friend can spend \$26 each day.

- x Press **EXIT** to return to the “Table Function” screen.
- x Use the down arrow cursor to highlight Y5. Use the right cursor arrow to highlight the 5 in 25X and change this 5 to a 6. Press **EXE** and **F6** to see the table.

Notice that the friend can spend \$26.00 and still go on a 21-day trip. Similar steps can be used to determine if the friend can spend \$27 each day. This time, -17 is shown in the table. Consequently, if the friend spends \$27.00 per day, there will not be enough money for a 21-day trip.

EQUATIONS IN ONE VARIABLE

AN ALTERNATE SOLUTION TO PROBLEM 1 USING GRAPHING

There are, of course, other techniques for investigating this problem. One good way is with a graph. You may wish to come back to this problem and explore it with graphs after students have been exposed to equations in two variables, but you also may wish to use this as an introduction to the topic. The solution below addresses the first part of the FIELD TRIP problem.

From the MAIN MENU, choose “Graph.” Then,

- x Delete all the functions that are there by highlighting them and pressing **F2** followed by **F1** .
- x We'll now set the viewing window. Press **SHIFT** **F3** to access the window. Type in appropriate values, pressing **EXE** after each value you type. So that you can clearly see the axes, you may wish to extend your domain and range slightly beyond the values you wish to explore. See below left for a possible window. Press **EXIT** when finished.
- x Enter the two functions in as Y1 and Y2, pressing **EXE** after you type in each. See below right.

```
View Window
Xmin  :-1
max   :16
scale:1
Ymin  :0
max   :600
scale:50
[INIT][TRIG][STD][STO][RCL]
```

```
Graph Func :Y=
Y1:425-30X
Y2:550-45X
Y3:
Y4:
Y5:
Y6:
[SEL][DEL][TYPE][CLR][MEM][DRAW]
```

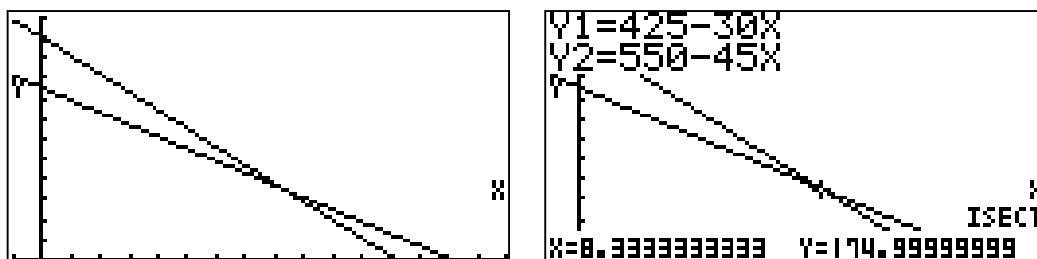
- x To see the graph, press **F6** . See below left.
- x To explore the point where the two lines meet, we can TRACE the functions to see approximately where they meet. To access the TRACE function, press **F1** . Then use the right and left arrow keys to move along a function and the up and down arrow keys to switch between functions.

EQUATIONS IN ONE VARIABLE

- x Once the cursor is close to the intersection, you can use the ZOOM function to get a closer look. To do this, after you have moved the cursor close to the point of intersection, press $\boxed{F2}$ for ZOOM and then $\boxed{F3}$ to ZOOM IN.
- x You can then use the TRACE feature again by pressing \boxed{SHIFT} $\boxed{F1}$.

If you are interested in a more accurate solution, the calculator can also help.

- x With the graph displayed but no functions shown, press $\boxed{F5}$ for the “Graph Solver.”
- x Press $\boxed{F5}$ for “Intersection” and wait a few seconds. The calculator shows the point of intersection. See below right.



This tells us, once again, that after a little over eight days into the trip, both will have \$175 remaining.

REFERENCE: *Addison-Wesley Secondary Math. An Integrated Approach. Focus on Algebra*, 1996.

EQUATIONS IN ONE VARIABLE

PROBLEM 2: PUBLIC TRANSPORTATION

Before you go to college, your family is planning to take a family vacation to the Grand Canyon. To save wear and tear on your family car, your parents decide to rent a car for the trip. U-Rent has two options for renting cars. For Option A, you must pay \$15.00 per day and \$0.15 per mile. For Option B, you must pay a flat fee of \$49.95 per day.

- A. On a map, locate your hometown and the Grand Canyon. Estimate the distance between your hometown and the Grand Canyon. How many miles do you estimate you will travel on the round trip?
- B. Write an expression for the cost of renting a car under Option A. Define any variables you use.
- C. Write an expression for the cost of renting a car under Option B. Define any variables you use.
- D. Write an equation that can be used to determine when the cost of Option A is equal to the cost of Option B.
- E. How many days will you have to travel until both plans cost the same amount?
- F. Which plan should you choose? Justify your reasoning.

EQUATIONS IN ONE VARIABLE

ONE SOLUTION TO PROBLEM 2: PUBLIC TRANSPORTATION

- A. On a map, locate your hometown and the Grand Canyon. Estimate the distance between your hometown and the Grand Canyon. How many miles do you estimate you will travel on the round trip?**

Answers vary. For the purpose of this solution, assume that the round trip is 1500 miles.

- B. Write an expression for the cost of renting a car under Option A. Define any variables you use.**

If we let X represent the number of days traveled, then the expression for the cost of a car under Option A is $15X + .15(1500)$.

- C. Write an expression for the cost of renting a car under Option B. Define any variables you use.**

Again letting X represent the number of days traveled, the expression for the cost of a car under Option B is $49.95X$.

- D. Write an equation that can be used to determine when the cost of Option A is equal to the cost of Option B.**

All we need do is set the two expressions equal. We have

$$15X + .15(1500) = 49.95X$$

- E. How many days will you have to travel until both plans cost the same amount?**

There are many ways to solve this. One method is with a table. From the MAIN MENU, call up the “Table” function.

x For Y1, type in $15X + .15(1500)$. Press EXE .

x For Y2, type in $49.95X$ and press EXE .

x Delete any other functions there by highlighting them and pressing F2 followed by F1 . See below left.

EQUATIONS IN ONE VARIABLE

x Next, press **F5** so we can set the range. One possible set of values is shown below right. Press **EXE** after each new value you type. See below right. Press **EXIT** when finished.

```
Table Func :Y=
Y1:15X+.15(1500)
Y2:49.95X
Y3:
Y4:
Y5:
Y6:
SEL DEL TYPE COL? RANG TABEL
```

```
Table Range
X
start:0
End :15
Pitch:1
```

x Press **F6** to see the table.

Students should note that Option A is less expensive for a trip lasting from 0 to 6 days, but that Option B is less expensive for any trip lasting 7 or more days. Since the cost will most likely be charged on a whole day basis, with partial days counting as whole days, this should help students determine which plan to select. Other students may choose the “Equation Solver” or the “Graph” features of the calculator to arrive at a solution of 6.4. Nevertheless, since rental car companies usually don't calculate costs for portions of a day, students realize that the plans will never cost the same, that their decision should be based on whether the trip will last seven or more days.

F. Which plan should you choose? Justify your reasoning.

Answers will vary as the mileage from your students' hometown to the Grand Canyon and the number of days suggested for the trip will differ.

EQUATIONS IN ONE VARIABLE

PROBLEM 3: THE PIGGY BANK

John has \$150.00 in his piggy bank and plans to add \$2.00 each week. Parneshia has \$200 in her piggy bank and plans to add \$6.00 dollars each week. After how many weeks will Parneshia have twice as much money as John?

REFERENCE: Mathematics Teaching in the Middle School, NCTM, Vol.2 No 6, May 1997, Menu of Problems.

PROBLEM 4: SWIM RECORDS

In 1988, the women's record for the 100-m freestyle in swimming was 54.73 seconds. It had been decreasing at a rate of 0.33 seconds a year. The men's record was 49.36 seconds and had been decreasing at 0.18 seconds a year. Assuming that these rates continue, after how many years will the records be the same? Do you think this will happen? Why or why not?

REFERENCE: UCSMP, Algebra, 1988.

EQUATIONS IN ONE VARIABLE

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	4.1, 4.2, 4.3
AWSM – Focus on Advanced Algebra (1998)	
Glencoe – Algebra 1 (1998)	1.5, 2.9, 3.1, 3.2, 3.5, 4.1, 4.4
Glencoe – Algebra 2 (1998)	1.4, 1.5A
Holt Rinehart Winston – Algebra (1997)	3.5, 4.4, 4.5
Holt Rinehart Winston – Advanced Algebra (1997)	1.5
Key Curriculum – Advanced Algebra Through Data Exploration	
Merrill – Algebra 1 (1995)	3.1, 3.2, 3.3, 3.5, 3.6
Merrill – Algebra 2 (1995)	1.3
McDougal Littell – Algebra 1: Explorations and Applications (1998)	2.1, 2.2, 2.3, 4.3, 4.4
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	1.4, 3.1, 3.2, 3.4, 3.5
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	3.1, 3.2, , 3.3, 3.4, 3.5
Prentice Hall – Algebra (1998)	3.1, 3.2, 3.3, 3.4, 3.5, 3.7, 4.1, 4.2, 4.4, 6.2
Prentice Hall – Advanced Algebra (1998)	2.4
SFAW: UCSMP – Algebra Part 1 (1998)	1.5, 2.6, 2.7, 3.5, 3.6, 4.3, 5.3, 5.8, 5.9, 6.1
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	1.5, 5.2
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern Algebra 1 – An Integrated Approach (1997)	3.1, 3.3, 3.4, 3.6, 3.7