CLEMSON ALGEBRA PROJECT
UNIT 3: LINEAR FUNCTIONS

PROBLEM 1: WALK BY GRAPH

For this investigation, you will need your graphing calculator, the EA-100, and the motion detector. Make sure the MOTION program, along with its subsidiary programs, has been loaded on your calculator. Then, call up the program and select "WALK BY GRAPH" and "Linear." Try to match the picture shown in your viewing window. Complete this activity several times. Discuss what you have discovered about the relationship between the graph and what you need to do to match it with your walk. What are the critical elements that make the graphs linear? Use your findings to develop a general form for linear functions.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

Casio EA-100 Data Collector

Pasco Motion Detector

EXTENSIONS

Again using "WALK BY GRAPH" on the MOTION program, select other function types and try matching these graphs. Explore the differences you find among the different function types.

ONE SOLUTION TO PROBLEM 1: WALK BY GRAPH

The MOTION program has been designed to help students to make connections among graphs, equations, and the real world, using the *x*-axis as time and the *y*-axis as distance. First, make sure that each calculator that will be used with the data collector has the series of six programs entitled “MOTION,” “ MOTION1,” “MOTION2,” “MOTION3,” “MOTION4”, and “MOTION5.” To transfer these programs from one calculator to another, make sure the calculators are connected with the supplied cable. Turn the calculators on, and

- Select “Link” from the MAIN MENU on both the receiving and sending calculators.
- Press [F2] on the receiving calculator.
- Press [F1] on the sending calculator.
- Press [F1] to select the desired programs.
- Use [F1] and the down arrow to select each of the six programs named above.
- Press [F6] to transmit.

If you get an error message, make sure the cable is securely attached to both calculators.

Before the students begin to work on the problem, the teacher should introduce the program with a demonstration for the entire class. First, all of the tools must be hooked up correctly.

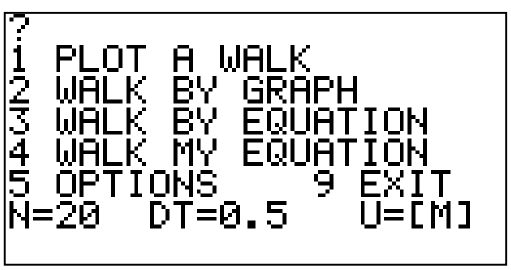
- Set the switch on the top of the motion detector on wide range. Depending on the physical arrangement of the room, this may have to be reset on narrow range. This will be necessary if the motion detector picks up the wrong things along the borders of the path.
- Clear a path about 2 meters wide and 6 meters long. This will be used for a person to walk toward and away from the motion detector.
- Using the cord, attach the calculator to the EA-100 data collector.
- Turn the EA-100 on.

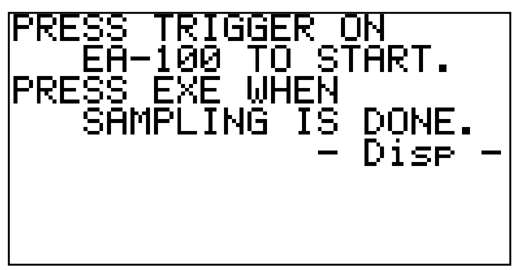
- Connect the EA-100 to the motion detector with the supplied cable. The cable plugs into the sonic port on the EA-100. Set the motion detector along the walking path at a height that will “hit” the walker.
- Turn the calculator on. From the MAIN MENU, call up “Program.”
- Highlight MOTION and press EXE .

Before working on the problem, students should familiarize themselves with the program. Look at the bottom of the MOTION menu screen (see below left). N=20 indicates that 20 data points will be collected. DT=0.5 means that the points will be collected every half second (DT stands for ‘delta time’). U=[M] tells us that the units are in meters. These values can be changed by selecting OPTIONS from the menu, but it is recommended to begin with the default values shown.

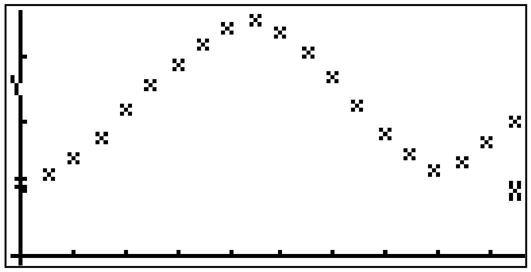
Although the problem asks students to explore “WALK BY GRAPH,” to become familiar with the program, begin with option 1, “PLOT A WALK.”

- From the menu screen shown below left, press 1 and EXE .
- Follow the directions, which tell you to press the TRIGGER key on the EA-100 when you are ready to start and the EXE on the calculator when the sampling is finished. See below right.





After you have finished, a graph of the walk should be displayed on your calculator. The screen below shows one particular walk. First, the teacher should point out that the tick marks on the *x*-axis represent seconds and the tick marks on the *y*-axis represent meters away from the detector. The teacher should then lead a class discussion helping students to interpret the graph.

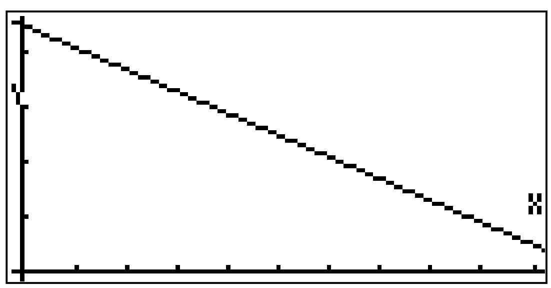


Some issues and questions that the teacher may wish the class to discuss follow.

- Let's make sure everyone understands what a point represents. For example, what does the second point to the right of the *y*-axis represent? (One second after the trigger was pressed, which started the data collection, the walker was about 1.5 meters away from the detector.)
- Which direction did the walker move when the program started? (Away from the detector.)
- How far from the detector did the walker start? (About 1 meter, because the graph appears to have a *y*-intercept about 1.)
- What direction is the walker moving when the graph goes up? (Away from the detector – the distance is becoming greater.)
- What direction is the walker moving when the graph goes down? (Toward the detector – the distance is becoming smaller.)
- When did the walker change directions? (Approximately 4.5 and 8 seconds into the data collection – this is based on when the graph changes from increasing to decreasing or decreasing to increasing.)
- What would a straight line represent? (Motion at a constant speed.)
- What would a curved line represent? (Motion at changing speeds.)
- Could you "walk" a circle? (No, you can't be in two places at the same time.)
- Are there shapes that cannot be "walked"? What are they? What do these shapes have in common? (Answers may vary. Key ideas are the positive values in the domain and range, and that for any input there is only one output – one definition for function.)
- What does the domain refer to in this setting? (The time – 0 to 10 seconds.)
- What does the range refer to in this setting? (Distance – 0 to about 4 meters.)

After one or two trials with "PLOT A WALK," the class should be ready to move to the second component of the MOTION program, "WALK BY GRAPH." Depending on the physical size of the room, the number of students, and, of course, the number of equipment set-ups available, divide the class into groups. Groups of three or four often work best, but groups of two can work too. Each person should be assigned roles. For example, for a group with four people, one person can be the "techie," assigned to make sure the equipment is hooked up properly and the path for the walker is clear. One person can be the "trigger person," giving the commands to the data collector and the calculator. A third can be the "commander," telling the walker where, when, and how fast to go. The fourth person can be the "walker." Group members should rotate through these roles.

To begin, from the MOTION menu, press [2] and [EXE] . We are interested in LINEAR functions, so press [1] and [EXE] . A graph, chosen randomly by the calculator within parameters set within the program, will appear. One such graph is shown below.



The group should discuss how to match this graph through their motion. For the graph shown above, the person should start about 4.5 meters away from the motion detector and then move towards the detector at a constant speed, reaching about .5 meters away from the detector in 10 seconds.

After the "commander" has indicated to the "walker" what to do, the "trigger person" should press [EXE] on the calculator. As before, the screen instructs the user to press the [TRIGGER] on the data collector to begin and [EXE] on the calculator after the data have been collected. Before pressing the [TRIGGER] , the group may note that the display on the data collector shows how far the person is away, so the starting point can be determined fairly accurately.

After EXE has been pressed, the calculator displays both the original graph and the results of the "walker." After a few trials, the group should become more adept at matching the graph. If desired, they can select the option to retry a particular graph.

NOTE: If for some reason a mistake is made while collecting data and the AC/ON key is pressed to break out of the program, you will need to unplug the EA-100 and reset the equipment as described in the beginning of the module.

By using this program, students should discover the key components of linear equations. The teacher should pose the following questions for students to reflect upon and answer and then lead a class discussion concerning the main considerations of linear functions in two variables.

- The *y*-intercept is the starting point, representing the distance the "walker" is from the motion detector at time 0.
- If the graph goes up, the distance from the motion detector increases. If the graph goes down, the distance from the motion detector decreases.
- Linear functions have constant slope. This means that the rate at which the "walker" moves must stay the same throughout the entire walk.
- The rate at which the "walker" should move can be determined by two points on the graph. In our example, at time 0, the "walker" was to be 4.5 meters away from the motion detector. After 10 seconds, the "walker" was to be 0.5 meters away. In other words, the "walker," starting 4.5 meters away from the motion detector, needed to move towards the detector at a steady speed, covering 4 meters in 10 seconds. This equates to 0.4 meters (40 cm) every second.

The teacher can use these ideas to build toward an intuitive understanding of $y = b + ax$ and then $y = ax + b$ forms for linear equations.

- At 0 seconds, the walker should be 4.5 meters away from the detector.
- At 1 second, the walker should be $4.5 - 0.4$ meters away.
- At 2 seconds, she/he should be $(4.5 - 0.4) - 0.4$ or $4.5 - 0.4 \times 2$ meters away.
- At 3 seconds, she/he should be $((4.5 - 0.4) - 0.4) - 0.4$ or $4.5 - 0.4 \times 3$ meters away.
- Finally, at *x* seconds, the walker should by $4.5 - .4x$ meters away.

PROBLEM 2: ARM LENGTH AND SHOE SIZE

Someone claims that there is a strong relationship between the length of people's forearms and their shoe sizes. You are to investigate this claim. So that males and females can be considered together, either subtract 1.5 from the women's shoe sizes or add 1.5 to the men's shoe sizes.

A. After gathering the data, construct a scatterplot. What domain, range, and scaling factors work well? Do length of forearm and shoe size appear to be related? What type of relationship seems appropriate?

B. How linear are the data? Perform linear regression and determine how well the regression line describes the data.

C. Interpret the slope of the regression line. Be sure to include units in your interpretation. Does this have real-world meaning? If so, what?

D. Interpret the y-intercept of your regression line. Be sure to include units in your interpretation. Does this have real-world meaning? If so, what?

EXTENSIONS

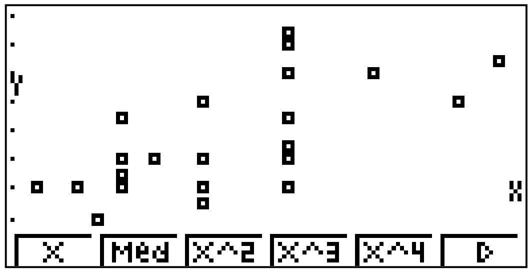
1. Find people outside of your classroom and determine how well the model predicts their shoe size when you know their arm length and how well the model predicts their arm length when you know their shoe size.
2. Investigate the residuals of the linear model. Is there a pattern? If so, can you predict what type of regression model might be more appropriate?
3. Explore other regression models on your calculator and find ones that appear to fit the data better. If so, can you come up with a theory that might support using that type of regression model?

ONE SOLUTION TO PROBLEM 2: ARM LENGTH AND SHOE SIZE

A. After gathering the data, construct a scatterplot. What domain, range, and scaling factors work well? Do length of forearm and shoe size appear to be related? What type of relationship seems appropriate?

The data below have been entered into Lists 1 and 2. (NOTE 1: if you wish to save previous data, after accessing the “List” menu, press SHIFT MENU to enter the SET UP, and choose any File that you have not yet used.) (NOTE 2: For consistency in units, 1.5 was subtracted from the females’ shoe sizes.) Using data from your own class is recommended. These data were from an exercise conducted by Dr. Linda Taylor at the University of Cincinnati.

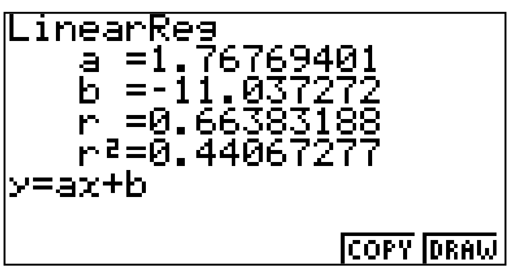
PERSON	ARM LENGTH (in.)	SHOE SIZE
1	10.5	7
2	11	11
3	10	7
4	10	8.5
5	11.5	10
6	10.5	6
7	9.875	5
8	11	7
9	11	10
10	11	11.5
11	9.75	6
12	11	7.5
13	11	6
14	10	7
15	10.5	5.5
16	11	8.5
17	10	6
18	9.5	6
19	12.25	10.5
20	10.2	7
21	10.5	9
22	11	8.5
23	10	6.5
24	12	9

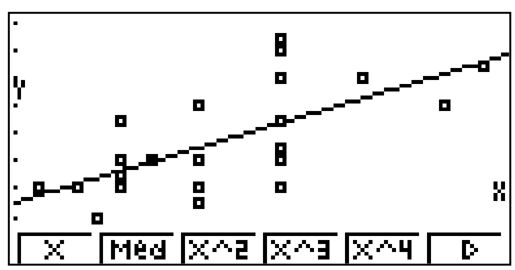


The scatterplot suggests a positive relationship, but this relationship does not appear to be very strong. Without a theory, the only model that is appropriate would be a linear model; at least this would suggest that as arm length increases, so too does shoe size.

B. How linear are the data? Perform linear regression and determine how well the regression line describes the data.

Again, the relationship does not appear to be strong. The regression equation, $y = 1.7677x - 11.0372$, which can be obtained by pressing [F1] from the scatterplot, has a correlation of .6638. Although this may be statistically significant (and is obviously positive), r^2 tells us that we can only reduce our error by 44.1% in predicting shoe size by knowing a person's arm length. The relationship may not be as strong as might have been expected.





C. Interpret the slope of the regression line. Be sure to include units in your interpretation. Does this have real-world meaning?

The slope is 1.7677. Since slope is the ratio of the change in the dependent variable (shoe size the way we have set up our data) to the change in the independent variable (forearm length), we can say that, on average, for every increase of one inch in arm length, the shoe size increases 1.7677 sizes. This interpretation does have meaning as stated, but keep in mind the relatively low correlation along with the limited domain.

D. Interpret the *y*-intercept of your regression line. Be sure to include units in your interpretation. Does this have real-world meaning? If so, what?

The *y*-intercept is the point on the graph that has an *x*-value of 0. For our regression equation, the *y*-intercept is (0, -11.0372), suggesting that a person with a forearm length of 0 should wear shoe size –11.0372! This, of course, is ridiculous. In many problems, the *y*-intercept does not have any significance simply because 0 is not part of a realistic domain.

In our problem, the *x*-values (the arm lengths) ranged from a low of 9.5 to a high of 12.25. Extrapolating much beyond this domain is risky at best.

It is also worth noting the small range of *x*-values. One cause for the relatively low correlation may be measurement errors. Exactly where does a forearm start and stop? How accurate do you believe these measures to be, especially when you consider that different people did the measuring?

PROBLEM 3: WALK BY EQUATION

Using the MOTION program on your calculator, select "WALK BY EQUATION," and choose linear equations. In this program, x represents the number of seconds you have been walking and y represents your distance from the motion detector. Try this several times. Then discuss the specific effects A and B have on the equation in the form $y = Ax + B$.

PROBLEM 4: A CIRCULAR RELATIONSHIP

Find at least five circles of various sizes. For each circle, measure the maximum distance across the circle and the distance around the circle.

A. First, investigate discrepancies in the measures within your group and determine a reasonable method of arriving at measures that are as accurate as possible.

B. Then, explore the relationship between the two variables and determine an appropriate mathematical model. Include a scatterplot and a regression equation.

C. Interpret your regression equation in your analysis, paying special attention to the meaning of the slope and the y-intercept.

D. Have you seen a formula similar to this somewhere? If so, where?

E. For what domain do you believe your model is appropriate?

F. Based on your interpretation of the model, answer the following question. Assume the distance around the equator is 25,000 miles. How many extra miles does an airplane fly on a trip around the equator if it flies at an altitude of 10 miles?

"BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

The ALGEBRA menu on the FX2.0 can be used to solve linear equations in a single variable.

- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation you desire and press [EXE] . For example, suppose you wish to solve the equation $2(x+3)-5(4x+7)=4(2x-8)$. Simply display on the top line the following: solve $(2(x+3)-5(4x+7)=4(2x-8))$. When you press [EXE] , the calculator displays $x=\frac{3}{26}$.

The "Algebra" menu can also be used to help students solve literal equations for *y* or any other variable.

- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press [EXE] . The top of the screen might look like this: solve $(4X+6Y=200,Y)$. The calculator returns $y=-\frac{2}{3}x+\frac{100}{3}$.

Using the Algebra Menu to Solve Manually

In addition to solving equations, the FX 2.0 can also assist students in learning the process for themselves. From the MAIN MENU, access the "Algebra" menu and follow the steps below. This shows an example with two variables, but equations in one variable can also be solved using the same technique.

- To clear other entries, press the function key for "Clear," the number for "ALL EQUATIONS," and [EXE] for yes.
- At the cursor, type in the equation you want to work on and press [EXE] . For example you might type $4X + 6Y = 200$. This equation is labeled as equation 1.
- Suppose we wanted to solve this equation for *Y*. All we need do to begin is simply tell the calculator to subtract 4*X*. The calculator assumes you are referring to the equation in its memory. Type in "minus 4*x*" in symbols and press [EXE] . This becomes equation 2, and should appear as $4x + 6Y - 4X = 200 - 4X$.
- To simplify equation 2, press [F1] for the TRNS menu, the number for simplify, and [EXE] . The result has isolated 6Y, showing us that $6Y = -4X + 200$. This is labeled as equation 3.
- The next step, of course, is to divide by 6. Simply press the division sign, 6 and [EXE] . The calculator shows $\frac{6Y}{6} = \frac{-4X + 200}{6}$.
- This needs to be simplified, so again press [F1] for the TRNS menu, the number for simplify, and [EXE] . The calculator shows that $Y = \frac{-2X}{3} + \frac{100}{3}$, using true fraction notation.

One of the many exciting features of the calculator is that it does whatever it is told to do, even if it does not help solve the problem. If the student tells the calculator to, say, multiply by 6 instead of divide by 6, the calculator does it, but the student can recognize that it does not help. The student can then return to the previous step and try something else.

Using the Tutorial Menu to Learn to Solve Linear Equations

The FX2.0 has a tutorial system built in that can help students to learn how to solve Linear Equations, Linear Inequalities, Quadratic Equations, and Simultaneous Equations. Within these, users can use equations already stored in the calculator or can input values for specific equations they wish to solve. The following demonstrates how the FX2.0 can help students solve the equation $-2x+5=4x+7$.

- From the MAIN MENU, highlight "Tutor" and press EXE .
- Use the down arrow to highlight $AX + B = CX + D$ and press F2 for "Input."
- Type in –2, 5, 4, and 7 for A, B, C, and D, respectively, pressing EXE after each.
- Press F6 twice and the calculator will set up an automatic, step-by-step solution.
- Use F6 and F1 to move forward and backward through the solution. At each step of the way, the calculator indicates what it should do.

These techniques can be very effective tools in helping students master the skills they need to solve equations and inequalities. By "busting this barrier" that impedes the progress of so many students, the ALGEBRA FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	3.2
AWSM – Focus on Advanced Algebra (1998)	2.1, 2.2
Glencoe – Algebra 1 (1998)	4.8, 5.3, 5.4, 5.6, 6.1, 6.3
Glencoe – Algebra 2 (1998)	2.2, 2.3, 2.5
Holt Rinehart Winston – Algebra (1997)	5.1, 5.2, 5.3, 5.4, 5.6
Holt Rinehart Winston – Advanced Algebra (1997)	1.2, 2.5
Key Curriculum – Advanced Algebra Through Data Exploration	4.1, 4.2, 4.5, 4.7
Merrill – Algebra 1 (1995)	9.4, 10.1, 10.2, 10.3, 10.4
Merrill – Algebra 2 (1995)	2.1, 2.6
McDougal Littell – Algebra 1: Explorations and Applications (1998)	2.5, 3.3, 3.4, 3.5, 3.6
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	4.2, 4.3, 4.4, 4.5, 4.6, 5.1, 5.4, 5.7, 11.2
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	8.2, 8.8
Prentice Hall – Algebra (1998)	2.1, 2.5, 5.1, 5.2, 5.3, 5.4, 5.6, 5.9
Prentice Hall – Advanced Algebra (1998)	1.1, 2.1, 2.2, 2.3, 2.5
SFAW: UCSMP – Algebra Part 1 (1998)	3.8, 4.6, 4.9, 5.5
SFAW: UCSMP – Algebra Part 2 (1998)	7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	3.1, 3.2, 3.3, 3.4, 3.5, 3.6
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	4.4, 4.5, 6.2, 6.3, 6.4, 6.5

