CLEMSON ALGEBRA PROJECT
UNIT 4: INEQUALITIES IN TWO VARIABLES

PROBLEM 1: GIFT GIVING

Gift-giving season is approaching and you wish to purchase gifts for four relatives and six friends. The amount you spend on each relative is usually, but not necessarily, different from the amount you spend on each friend. You have saved your money and can spend any amount up to $200. How much should you spend, on average, for each relative, and how much, on average, should you spend on each friend?

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

1. Change the amount of money from $200 to $100 and then to $500. What happens to the graph as you make these changes? Why?
2. Leave the amount at $200, but change the number of relatives. What happens to the graph as you make these changes? Why?
3. Leave the amount at $200, but change the number of friends. What happens to the graph as you make these changes? Why?
4. Compose some general rules that show how the graph of $ax + by \leq c$ is affected by *a*, *b*, and *c* respectively.
5. Investigate the problem if you were to spend the money on 4 close relatives, 6 distant relatives, and 8 friends.

ONE SOLUTION TO PROBLEM 1: GIFT GIVING

This problem can be addressed any number of ways. Two reasonable approaches include using a table and using a graph. In either case, however, some algebra simplifies the problem.

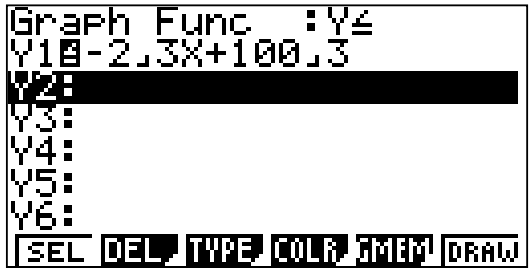
Let x represent the average amount spent on each relative and y represent the average amount spent on each friend. Because you have four relatives for whom you will buy something, $4x$ represents the amount you will spend on relatives. Similarly, $6y$ represents the amount you will spend on friends. You can spend any sum up to and including \$200, assuming any tax has already been included in the expressions listed above. Our relationship can thus be described by the inequality $4x + 6y \leq 200$.

Let's begin with a graph. To enter the relation into the calculator, we must first solve for y. First, subtract $4x$ from both sides to obtain $6y \leq -4x + 200$. Then, divide both sides by 6, obtaining $y \leq \frac{-2}{3}x + \frac{100}{3}$. (NOTE: If we chose not to simplify, we could use $y \leq (-4x + 200) \div 6$.)

Now we'll graph it. From the MAIN MENU, call up the "Graph" menu.

- If relations from previous work are entered, either delete them (highlight them and press [F2] followed by [F1]) or turn them off (highlight them and press [F1] so they are no longer selected). The following assumes that Y1 is available for the problem.
- To enter our relation, we first want to enter the correct type of relation. With Y1 highlighted, press [F3] for TYPE, [F6] for more options, and [F4] for the less than or equal to relation. This returns you to Y1.
- Simply type in the relation, using the [a b/c] key for the two fraction bars and press [EXE]. (NOTE: If the division symbol is entered instead, either enclose the $-2 \div 3$ in parentheses or put a multiplication symbol between the 3 and the x. If not, the calculator will treat the variable term as $\frac{-2}{3x}$ instead of $\frac{-2}{3}x$.)

The entry screen is shown below.



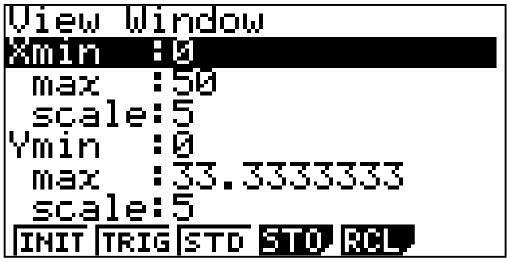
After entering, the relation, press F6 to view the graph. Assuming a standard window, which uses a domain and range from –10 to 10, the entire graph will be shaded in! This is due to an inappropriate window.

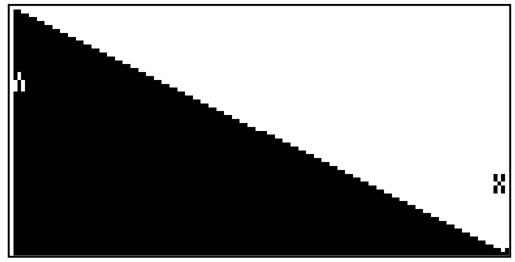
We'll fix the problem by determining a reasonable domain and range. First, discuss with students a reasonable domain for *x*. Keep in mind that *x* represents the average price of each gift for relatives. Obviously, it doesn't make sense to make the average negative, so a reasonable minimum value for *x* is 0. Because there are four family members and a maximum of $200 to spend, if the person decides to spend all of the money on relatives, the average still cannot exceed $50, so 50 is a reasonable maximum. Setting the scale at 5 may be an agreeable number of tick marks on the *x*-axis.

When thinking about the range for *y*, students should also realize that 0 is a minimum value to use. Because there are six friends to buy gifts for, the maximum the average could be is 200/6 or $33.33, rounded to the penny. If students wish, they can use 200/6 as the maximum value for *y*, or they may wish to round to perhaps 35. A scale of 5 again is appropriate, although students may prefer a different value.

To enter these values, when looking at the graph,

- Press SHIFT F3 to access the viewing window.
- Type in the numbers discussed above.
- Press EXE after each entry. After the last entry, press EXIT to return to the input screen.
- Pressing F6 will draw the graph with the new window. The window and graph are shown below.





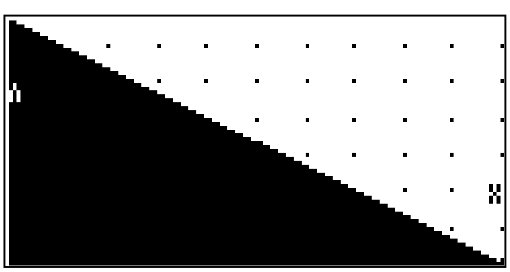
An alternate method of setting up an appropriate viewing window is to input the desired values for *x* as explained above and drawing the graph. Then,

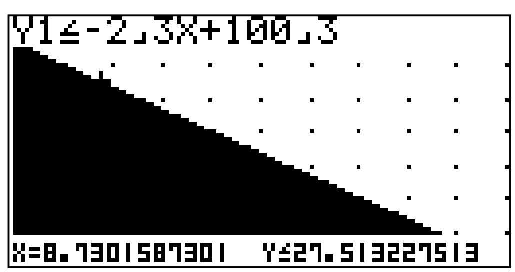
- Press F2 to obtain the ZOOM window.
- Press F5 for an "AUTOMATIC" window determined by the calculator.

This ZOOM AUTO feature can be very helpful when you have determined a reasonable domain, but wish help in determining a reasonable range.

Although it does not transfer well to paper as shown below, turning on the grid lines may make it easier for students to read and interpret the graph. Because we set both scales at 5, both horizontal and vertical moves of one dot represent a change of $5. To turn on the gridlines, with the graph displayed,

- Press SHIFT MENU to reach the graph set-up screen.
- Then, use the down arrow until GRID is highlighted.
- Press F1 to turn the grid on and EXIT followed by F6 to return to the graph. (See below left.)
- The Trace feature, accessed by pressing F1 , can help students recognize the vast number of possible solutions. See below right.





Depending on their maturity and background, students may have expected only one specific answer when you first presented the problem. Obviously, that is not the case; several values are possible for the mean cost of a relative's present and the mean cost of a friend's present. After they have studied the graph and traced through the boundary line, ask them to identify points in the shaded region, reminding them that the scale has been set at five. As they create their list, they should keep in mind what the point means. For example, the point (20, 15) is in the shaded region. This means that to keep within the $200 limit, it is okay to average $20 per gift for each relative and $15 per gift for each friend.

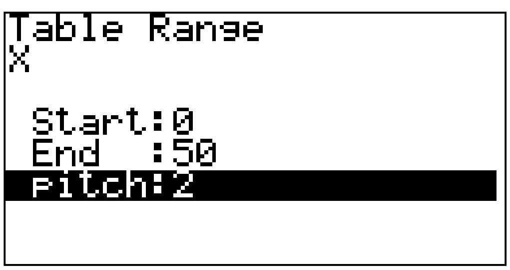
Then ask other questions, such as the following:

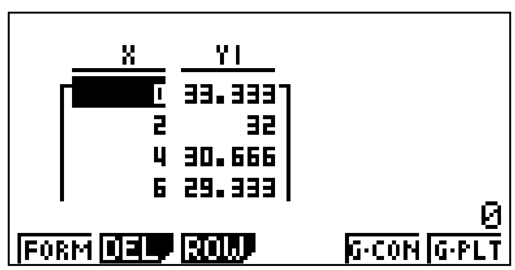
- What is the significance of moving to the right within the shaded region? (Because the *x*-value is increasing, this means they are spending more on average for each relative gift.)
- What is the significance of moving up within the shaded region? (This represents increasing the average cost of a friend's gift.)
- Are there values that indicate spending exactly $200? (With this scale, four points, (5, 30), (20, 20), (35, 10), and (50, 0), are on the line that defines the region. Because they are on this boundary, they indicate that the maximum amount available, $200, was spent.)
- Based on your graph, what would you do? Support your answer. (Answers will vary.)

ANOTHER TECHNIQUE USING A TABLE

Another way to explore the problem is with a table, looking for the maximum values for *x* and *y*. Once again the calculator can provide an excellent means for investigating. From the MAIN MENU, call up the "Table" function. To look at the table, we need an equation, not an inequality.

- To change the type of relation, press F3 for type and F1 for Y=.
- With Y1 highlighted, type in $\frac{-2}{3}x + \frac{100}{3}$, again using the a b/c to enter the fraction bar.
- The next step is to set the range for our *x*-values. Pressing F5 allows us to choose the values of *x* we would like to investigate. Because *x* can vary from 0 to 50, we should set 0 as our Start value and 50 as our End value.
- The pitch is up to us. If we want, we could look at every possible monetary value for *x* by entering a pitch of .01. This seems a little extreme, but some students may wish to find a solution that is this precise. Alternatively, we can check each dollar by entering a pitch of 1, or perhaps we could check fewer values by entering a pitch of 2 or even 5.
- Below left shows a pitch of 2 and below right shows the beginning of the table, obtained by pressing EXIT and F6 after the range was set.





Use the down and up arrows to scroll through the table, noting the possible maximum values for *x* and *y*. These are average values for relatives' and friends' gifts in which all $200 is spent; any values less than these would, of course, spend less than the allotted $200.

PROBLEM 2: MAKING THE GRADE

Your teacher tells you that your course grade will be determined by quizzes and tests, with your quiz average accounting for 40% of your grade and your test average for the remaining 60% of your grade. Explore the averages you need on quizzes and tests to ensure that you will have at least 80% for your course grade.

EXTENSION

Investigate the same problem, but make sure your quiz and test averages do not exceed 100%.

ONE SOLUTION PROBLEM 2: MAKING THE GRADE.

Our first order of business is identifying variables to represent the unknown quantities we wish to investigate.

- Let x represent the quiz average for the course.
- Let y represent the test average for the course.

Because of the weights put on these and our wish to have an average of 80 or higher, we can write an inequality as follows:

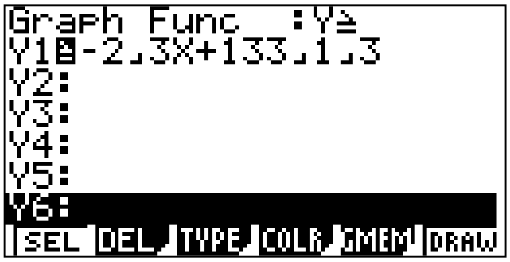
$$.40x + .60y \geq 80$$

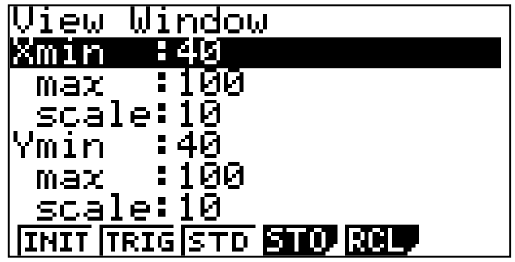
In the inequality above, we could have chosen to use .80 instead of 80, with x and y being expressed as decimals or percents. We have chosen, instead, to use scores such as 90 instead of 90%. This is an arbitrary decision, but one that should be made consciously.

We now wish to explore the relationship between x and y. To isolate y, we need to subtract .40x and then divide by .60. If we do not worry about simplifying, we have $y \geq (80 - .40x) \div .60$.

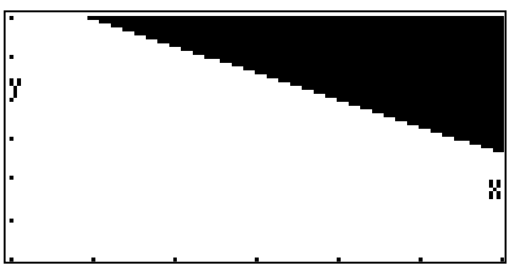
We will first explore this with a graph.

- From the MAIN MENU, choose "Graph."
- Delete or de-select any functions already shown ([F2] followed by [F1] to delete a highlighted function or just [F1] to de-select a highlighted function that is already selected.)
- Choose TYPE Y≥ by pressing [F3] , [F6] , and [F3] .
- Type in $\frac{-2}{3}x + 133\frac{1}{3}$ after Y1. Use the fraction key, [a b/c] , for the fraction and mixed number. See the screen below left.
- To set an appropriate window, press [SHIFT] [F3] and type in appropriate values, pressing [EXE] after each entry. One possibility is shown below right.





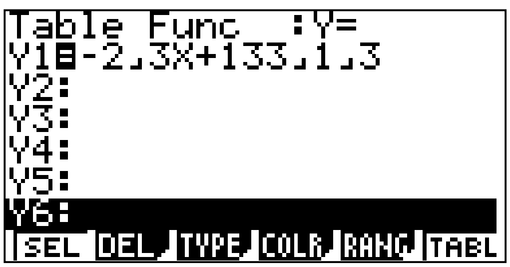
- After the window has been entered, press EXIT and F6 to view the graph. See below left. If you desire, press SHIFT MENU to access the set-up to display the grid (or to turn it off). Shown below is the graph with the grid off.
- If desired, you can trace along the border by pressing F1 and then using the right and left arrows as appropriate. Any point on or above this border will produce the desired result, an average of 80 or above. One such point, which can be rounded to (57.1, 95.2), is shown below right. This indicates that if we have a quiz average of 57.1 and a test average of 95.2, we will have an overall average of approximately 80. Higher values in either or both will result in a higher overall average.

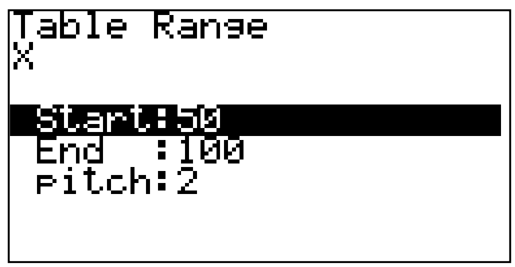


We may also explore our relation algebraically or numerically. Simplifying our inequality, we obtain $y \geq \frac{-2}{3}x + 133\frac{1}{3}$. One point to consider is the *y*-intercept. If *x* is 0, then *y* needs to be more than $133\frac{1}{3}$. Consequently, we cannot have an average of 0 on quizzes and expect to achieve an overall average of 80, assuming that 100 is the maximum test average. We can explore different points, perhaps substituting 100 for *x* (our quiz average) and determining what our test average needs to be. Similarly we can plug in 100 for *y* (our test average) to determine what *x*, our quiz average, needs to be.

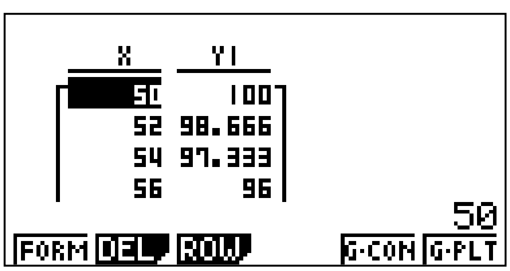
If we investigate the equation instead of the inequality, we can investigate our minimum requirements via a table. To do so, from the MAIN MENU, choose "Table." Then,

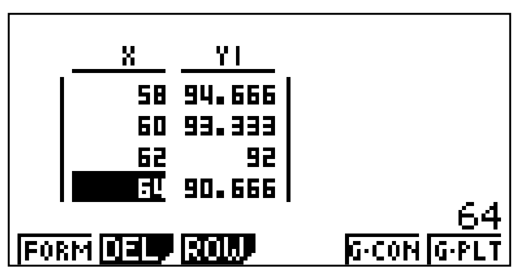
- Delete or de-select any functions already shown ([F2] followed by [F1] to delete a highlighted function or just [F1] to de-select a highlighted function that is already selected.)
- Choose TYPE Y= by pressing [F3] and [F1] .
- Type in $\frac{-2}{3}x+133\frac{1}{3}$ after Y1. Use the fraction key, [a b/c] , for the fractions, including separating the 133 from the fraction. See the screen below left.
- Press [F5] to set the range. Shown below right is a range of *x* from 50 to 100, with increments of 2 points each time.





- Press [F6] to view the table. Use the down and up arrows as desired to scroll through the table. The first few values are shown below.





This tells us a great deal. For example, the first point tells us that to achieve an 80, if our quiz average, *x*, is 50, we need a test average of 100. For a quiz average of 64, we need a test average of almost 91. We might note that for every increase in 2 points in our quiz average, our test average can decrease by one-and-one-third points. Half of one-and-one-third is two-thirds. Note that the slope of our equation is $\frac{-2}{3}$. In other words, for every increase of one point in quiz average, our test average can drop two-thirds of a point. Similarly, for every three-point increase in quiz average, our test average can drop two points.

Perhaps we may wish to think of the slope in another way. We can associate the negative sign with either the numerator or denominator. For example, we may choose to think of the slope as $\frac{4}{-6}$, which, of course, is equivalent to $\frac{-2}{3}$. If so, we can interpret our slope as saying that if we increase our test average by 4 points, we can let our quiz average drop 6 points. Using the table may be very helpful in discovering such relationships.

PROBLEM 3: PEANUT BUTTER AND JELLY

Eric loves peanut butter and jelly, sometimes together and sometimes separate. Peanut butter has 190 calories per serving, and grape jelly has 50 calories per serving. How many servings of each can Eric have if he keeps his total calorie intake from these two foods under 1000?

PROBLEM 4: CATS AND DOGS

Our veterinarian charges $55 for a check-up and the appropriate shots for each dog she sees. She charges $50 for similar work for each cat. How many dogs and cats should she schedule if she wishes to bring in at least $400 from these appointments?

"BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

The "Algebra" menu on the ALGEBRA FX2.0 can be used to help students solve equations and inequalities.

- From the MAIN MENU, highlight "Algebra" and press [EXE].
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation or inequality you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press [EXE]. To obtain a greater than or less than sign while typing, press the appropriate function key for "EQUATION," the number for "INEQUALITIES," and the number of the desired sign. The top of the screen might look like this: solve$(4+6Y \leq 200, Y)$. NOTE: The FX2.0 can solve literal equations, but not literal inequalities for a specified variable. This is to avoid division when the sign of the divisor is unknown.

Using the Algebra Menu to Solve Manually

In addition to solving inequalities, the FX 2.0 can assist students in learning the process for themselves. From the MAIN MENU, access the "Algebra" menu. Then,

- To clear other entries, press the function key for "Clear," the number for "ALL EQUATIONS," and [EXE] for yes.
- At the cursor, type in the equation or inequality you want to work on and press [EXE]. To obtain a greater than or less than sign, press the function for "EQUATION," the number for "INEQUALITIES," and the number of the desired sign. For example you might type in $4X+6Y \leq 200$. This inequality is labeled as equation 1.
- Suppose we wanted to solve this inequality for *Y*. All we need do to begin is simply tell the calculator to subtract 4*X*. The calculator assumes you are referring to the equation in its memory. Press [EXE]. This becomes equation 2, and should appear as $4x+6Y-4X \leq 200-4X$.

- To simplify equation 2, press F1 for the TRNS menu, the number for simplify, and EXE . The result has isolated 6Y, showing us that $6Y \leq -4X + 200$. This is labeled as equation 3. One of the many exciting features of the calculator is that, even if the student tells the calculator to do something wrong, the calculator does it, but the student can recognize that it does not help lead to a solution. The student can then return to the previous equation and try something else.
- The next step, of course, is to divide by 6. Simply press the division sign, 6 and press EXE . The calculator shows $\frac{6Y}{6} \leq \frac{-4X + 200}{6}$.
- This needs to be simplified, so again press F1 for the TRNS menu, the number for simplify, and EXE . The calculator shows that $Y \leq \frac{-2X}{3} + \frac{100}{3}$, using true fraction notation.

Using the Tutorial Menu to Learn to Solve Linear Inequalities

The FX2.0 has a tutorial system built in that can help students to learn how to solve Linear Equations, Linear Inequalities, Quadratic Equations, and Simultaneous Equations. Within these, users can use equations already stored in the calculator or can input values for specific equations they wish to solve. The following demonstrates how the FX2.0 can help students solve the equation $-2x + 5 \leq 4x + 7$.

- From the MAIN MENU, highlight “Tutor” and press EXE .
- Press F5 to change the type of inequality, followed by the appropriate number.
- Use the down arrow to highlight $AX + B \leq CX + D$ and press F2 for “Input.”
- Type in –2, 5, 4, and 7 for A, B, C, and D, respectively, pressing EXE after each.
- Press F6 twice and the calculator will set up an automatic, step-by-step solution.

- Use F6 and F1 to move forward and backward through the solution. At each step of the way, the calculator indicates what it should do.

These techniques can be very effective tools in helping students master the skills they need to solve inequalities. By "busting this barrier" that impedes the progress of so many students, the Algebra FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	6.3
AWSM – Focus on Advanced Algebra (1998)	2.3
Glencoe – Algebra 1 (1998)	7.8
Glencoe – Algebra 2 (1998)	2.7
Holt Rinehart Winston – Algebra (1997)	6.5
Holt Rinehart Winston – Advanced Algebra (1997)	1.6, 4.8, 4.9
Key Curriculum – Advanced Algebra Through Data Exploration	
Merrill – Algebra 1 (1995)	9.4
Merrill – Algebra 2 (1995)	2.8
McDougal Littell – Algebra 1: Explorations and Applications (1998)	7.5
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	6.2, 6.5
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	10.7
Prentice Hall – Algebra (1998)	6.5, 6.6
Prentice Hall – Advanced Algebra (1998)	2.5
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	7.9
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	5.7
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	8.1