CLEMSON ALGEBRA PROJECT
UNIT 5: LINEAR SYSTEMS

PROBLEM 1: NUTRITIOUS SNACKS

In an attempt to eat healthier, many teens are changing the amounts and types of foods that they eat. Recommended kilocalorie, abbreviated Calorie, intake for teens is 2200 - 2300 Calories per day. Most of these Calories should come from meals; however snacks are also important. Suppose that a teenager chooses to limit snacks to 500 Calories per day. Further suppose that carbohydrates from snacks should be limited to 88g and that the fat intake from snacks should be limited to 15g. If snacks are limited to microwave popcorn, sun-dried raisins and dry roasted peanuts, determine how much of each food a teenager can consume while remaining within the suggested guidelines.

A. Using the nutrition labels on page 2, calculate the Calories, carbohydrates, and fat for 1 gram of each snack food.

B. Determine the number of grams of popcorn, peanuts, and raisins that would provide 500 Calories, 88g of carbohydrates and 15g of fat.

C. Do you think that these foods are healthy choices for snacks? Explain.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

1. Select other nutrients and calculate the daily requirements, and decide how much of each nutrient should come from meals and snacks. Determine how much of each of the snack foods listed above should be eaten daily.
2. Calculate the *ideal* nutritional composition of each meal and snack. Maintain a nutrition journal for a week and compare the nutritional value of each meal and snack and compare your diet to an *ideal* diet.
3. Choose three snack foods that you regularly eat. Using the constraints listed in the original problem, determine how much of each food you can eat.

The tables below shows the nutritional information listed on the identified foods. Formatting has been changed for this document. Some information has been omitted.

MICROWAVE POPCORN				**UNSALTED DRY ROASTED PEANUTS**	
Nutrition Facts Serving Size about 3 Tbsp (37g) Unpopped (makes about 6 cups popped) Serving Per Bag about 2 Servings Per Package about 16				**Nutrition Facts** Serving Size 1 oz (28g/About 39 Pieces) Servings Per Container 16	
Amount Per Serving	3Tbsp Unpopped	6Cups (30g) Popped	1 Cup Popped	**Amount Per Serving**	
Calories	125	100	15	**Calories** 160 Calories from Fat 120	
Calories from fat	25	20	0		
	% Daily Value *			**% Daily Value ***	
Total Fat 3g	5%	3%	0%	**Total Fat** 14g	21%
Cholesterol 0mg	0%	0%	0%	**Cholesterol** 0mg	0%
Sodium 0mg	0%	0%	0%	**Sodium** 0mg	0%
Total Carbohydrate 25g	8%	7%	1%	**Total Carbohydrate** 6g	2%
Protein 4g				**Protein** 8g	8%
* percent Daily Values are based on a 2,000 calories diet. Your daily values may be higher or lower depending on your calorie intake.				* percent Daily Values are based on a 2,000 calories diet. Your daily values may be higher or lower depending on your calorie intake.	

CALIFORNIA SUN-DRIED RAISINS	
Nutrition Facts Serving Size 1 box (14.1g) Servings Per Container 14	
Amount Per Serving	
Calories	45
% Daily Value *	
Total Fat 0g	0%
Sodium 0mg	0%
Total Carbohydrate 11g	4%
Protein 0g	
* percent Daily Values are based on a 2,000 calories diet. Your daily values may be higher or lower depending on your calorie intake.	

ONE SOLUTION TO PROBLEM ONE: NUTRITIOUS SNACKS

A. Using the nutrition labels on page 2, calculate the Calories, carbohydrates, and fat for 1 gram of each snack food.

To determine the Calories, carbohydrates, and fat per gram, we need to divide the number of Calories, carbohydrates, and fat in each snack by the number of grams. For popcorn, using the information on 6 cups, the amount of Calories, carbohydrates, and fat are 100, 25, and 3, respectively, which we will divide by 30, the number of grams in the six cups. For the dry roasted peanuts, the amounts are 160, 6, and 14, all of which will be divided by 28. For raisins, the values are 45, 11, and 0, all of which will be divided by 14.1.

From the MAIN MENU, choose "Run." Using the values given above, simply type in the operation, pressing [EXE] after each entry. The results, rounded to two decimal places, are shown in the table below.

Snack Food	Calories/g	Carbohydrates/g	Fat/g
Popcorn (Popped)	3.33	0.83	0.1
Peanuts	5.71	0.21	0.5
Raisins	3.19	0.78	0

B. Determine the number of grams of popcorn, peanuts and raisins that would provide 500 Calories, 88g of carbohydrates and 15g of fat.

Using the information from the table above, we will write a system of linear equations.

Let p = the number of grams of popcorn we should have

n = the number of grams of peanuts we should have

r = the number of grams of raisins have.

Our system becomes:

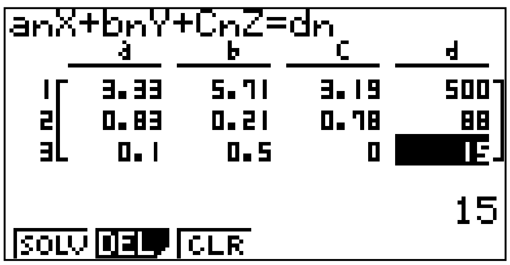
$$3.33p + 5.71n + 3.19r = 500$$

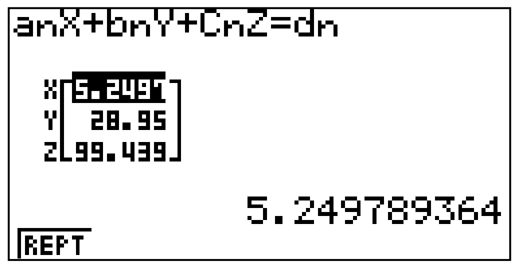
$$0.83p + 0.21n + 0.78r = 88$$

$$0.1p + 0.5n + 0r = 15$$

To solve this system of equations, from the MAIN MENU, select "Equations." Then,

- Choose F1 for "Simultaneous."
- Press F2 to indicate there are 3 unknowns.
- Type in the values from the system into the matrix, pressing EXE after each entry. Your screen should look like the one shown below left.
- To solve the system, after entering all of the values, press F1 . See below right.





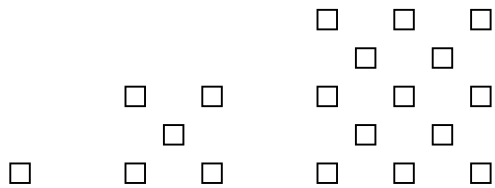
Although we used, *p*, *n*, and *r* instead of *X*, *Y*, and *Z*, our results tell us that *p*= 5.24 grams of popcorn, *n* = 28.95 grams of nuts, and *r* = 99.44 grams of raisins. Based on the serving sizes listed on the packaging, the snacks could consist of approximately 1 cup of popped popcorn (one serving is about 5 grams), 1 serving of dry roasted peanuts (one serving is about 28 grams), and 7 small boxes of raisins (one serving is about 14 grams).

C. Do you think that these foods are healthy choices for snacks? Explain.

Answers will vary.

PROBLEM 2: GEOMETRIC NUMBER PATTERS

Consider the following sequence of geometric number patterns and the number represented by each term.



1 5 13

A. Use color tiles to build this pattern. Build the 4th term in this sequence. How many tiles did you use?

B. How many tiles would you need to represent the 5th term?

C. Create a table relating the figure number with the number of tiles.

D. Graph the information from the table. Describe the graph.

E. Find a pattern in the table of values. In order to help discover the pattern, investigate the first order differences of the function values.

F. Investigate the second order differences.

G. Using the general form for a quadratic function, set up a system of equations from your table of values.

H. Solve this system of equations to find the quadratic function that describes this geometric number pattern.

I. Find the number of tiles needed to build the tenth term of this sequence.

EXTENSIONS

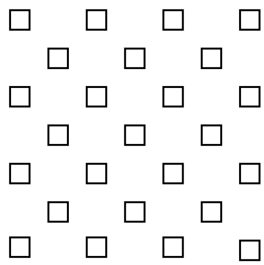
1. If two different colored tiles are to be alternated, determine the number of tiles of each color needed to cover the floor of a room.
2. Determine the number of tiles required for a pattern of this sort in a garden.

REFERENCE: *Mathematics for Elementary Teachers: An Activity Approach* by Albert B. Bennett, Jr / L. Ted Nelson. WCB McGraw - Hill, 1998.

ONE SOLUTION TO PROBLEM 2: GEOMETRIC NUMBER PATTERNS

A. Use color tiles to build this pattern. Build the 4th term in this sequence. How many tiles did you use?

The pattern should look like the one below. It takes 25 tiles.



B. How many tiles would you need to represent the 5th term?

So far the sequence has been 1, 5, 13, and 25. If we build the 5th set, we find that it takes 41 tiles.

C. Create a table relating the figure number with the number of tiles.

Figure Number (*n*)	Number of Tiles (f (*n*))
1	1
2	5
3	13
4	25
5	41

D. Graph the information from the table. Describe the graph.

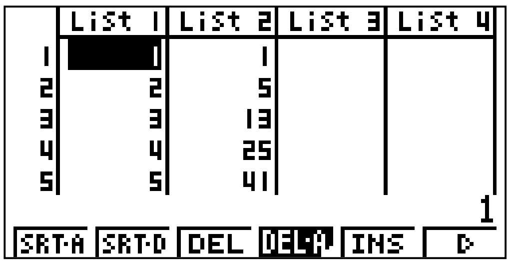
From the MAIN MENU, call up the "Statistics" menu. Then,

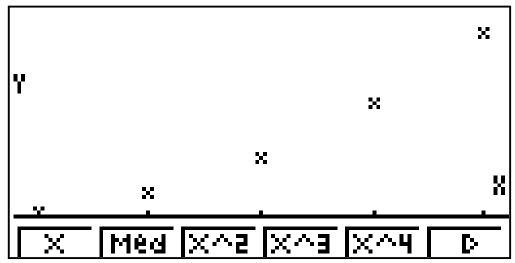
- To clear Lists 1 and 2: Highlight a value in list 1, press F6 for more options, F4 to delete all items, and F1 to confirm the deletion. Use the right arrow to move into List 2 and repeat the process.
- Enter the figure number into List 1, pressing EXE after each entry.
- Enter the number of tiles into List 2, again pressing EXE after each entry. See below left.

Next, we'll make sure the window is set to "automatic."

- Press [SHIFT] [MENU] to get to the Set Up.
- If "Stat Window" is not on automatic, press [F1] . Then press [EXIT] .

To view the scatterplot, press [F1] twice. See below right.

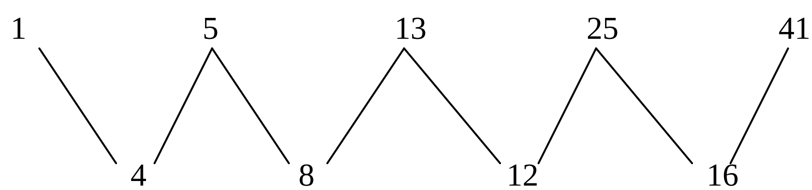




Note that the graph does not appear to be linear. It appears that the graph might be a portion of a polynomial, perhaps quadratic.

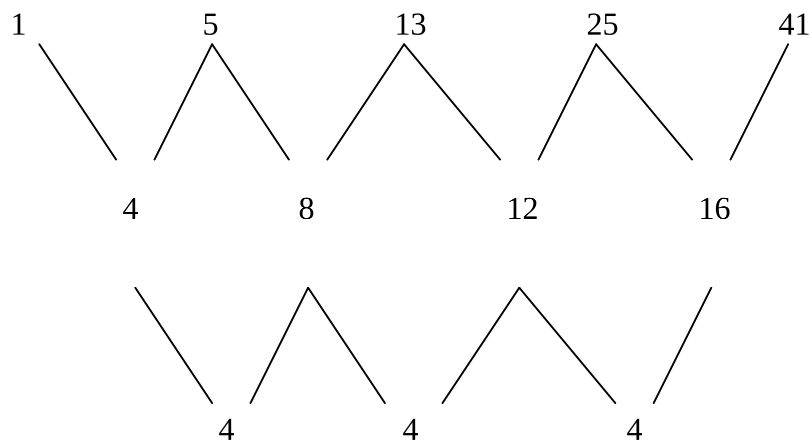
E. Find a pattern in the table of values. In order to help discover the pattern, investigate the first order differences of the function values.

Upon initial inspection, no pattern may be apparent, other than that the values are increasing. However if we look at the differences in the function values in List 2, we do find a pattern. The differences are {4, 8, 12, 16}.



F. Investigate the second order differences.

The second order differences refer to the differences of the first order differences. We can readily see that each value is increasing by 4.



Because the second order differences are constant, the function that describes this sequence is a quadratic polynomial.

G. Using the general form of a quadratic function, set up a system of equations from your table of values.

The general form for quadratic functions is $y = Ax^2 + Bx + C$. Substituting x and y values from our first three points, we obtain the following system:

$$A + B + C = 1$$

$$4A + 2B + C = 5$$

$$9A + 3B + C = 13$$

H. Solve this system of equations to find the quadratic function that describes this geometric number pattern.

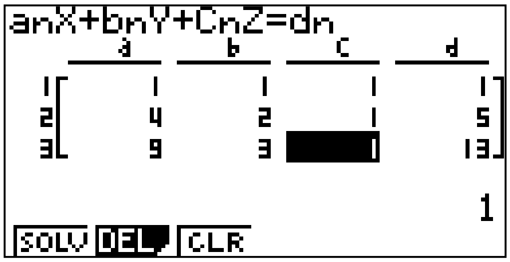
See the following page for two methods of solution. In either case, we find that $A = 2$, $B = -2$, and $C = 1$. Our function, therefore, is $y = 2x^2 - 2x + 1$.

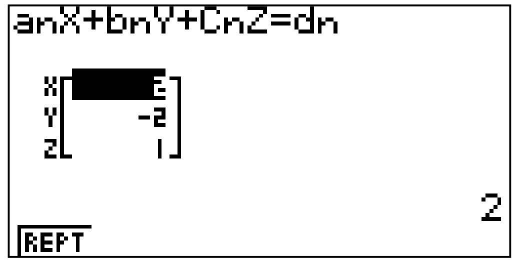
I. Find the number of tiles needed to build the tenth term of this sequence.

Substituting 10 for x tells us we'll need 181 tiles to build the 10^{th} term.

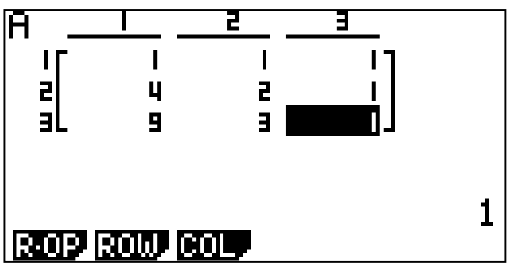
TWO SOLUTION METHODS FOR PART G

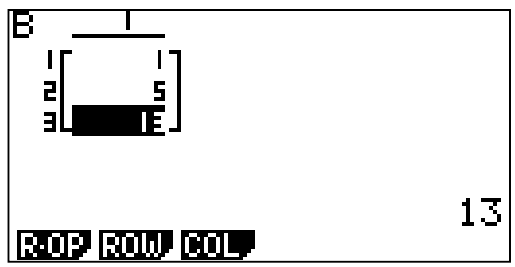
To solve the system of equations presented in part G, we could choose any of several methods. Two are shown here. Choose the “Equation” function from the MAIN MENU and select “Simultaneous.” The set up and solution look like the one below.





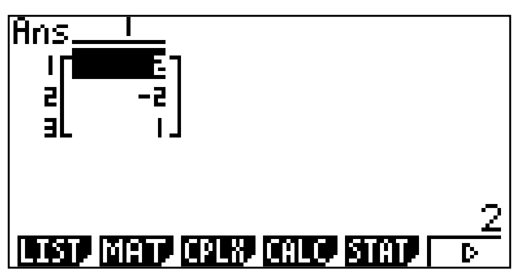
Alternately, we might solve with inverse matrices. First, choose the “Matrix” option from the MAIN MENU to create Matrices A and B. Set up Matrix A as the coefficient matrix (see below left) and Matrix B as the solution matrix (see below right).





We now want to find the product of the inverse of Matrix A and Matrix B.

- From the MAIN MENU, choose “Run.”
- Press [OPTN] [F2] for MATRIX, [F1] for Matrix and type in A
- Obtain the inverse by pressing [SHIFT] and the right parenthesis.
- Then press [F1] for Matrix followed by B. See below left.
- Press [EXE] for the solution. See below right.



PROBLEM 3: COMPUTER AIDED TOMOGRAPHY (CAT SCANS)

Computer Aided Tomography (CAT) scanners compute a picture of the inside of the body by making use of a coordinate grid and evaluating the mass of objects that can only be measured indirectly with X-rays. The basic principle behind CAT technology is that the intensity of an X-ray passing through a body is decreased by an amount proportional to the mass of the tissue and the distance the X-ray travels. As X-rays travel through the body, the change in intensity is measured and masses with different densities are located.

CAT scanners report measurements in units called *linear attenuation units* (LAUs). These units are logarithms of ratios of input energy to output energy. As an X-ray passes through grid cell A, the X-ray is weakened by a LAU. As the X-ray passes through grid cell B, it is weakened by b LAU. The total attenuation when the X-ray passes through both grids is $a + b$.

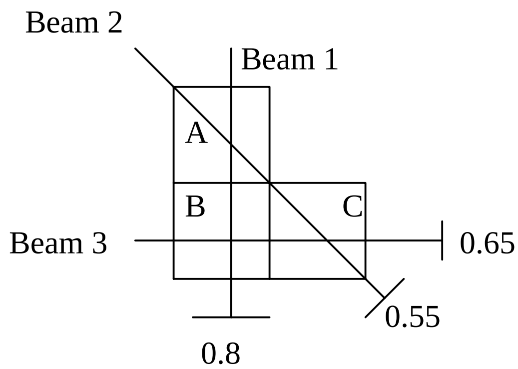
Ranges of LAU values have been established for healthy tissue, tumorous tissue, and bone. Note that there is overlap in Linear Attenuation Units between substance categories. This overlap is an indication that some CAT scan results may be inconclusive.

CAT Scanner Ranges

Substance	Linear Attenuation Values
Healthy Tissue	0.16225 — 0.2977
Tumorous Tissue	0.2679 — 0.3930
Bone	0.3857 — 0.5108
Metal	1.54 — ∞

Today, CAT scan machines use 512 × 512 grids which result in extremely complicated systems of equations. For the sake of simplicity, this activity deals with much smaller grids.

Consider the following example:



Suppose that as Beam 1 passes through grid cells A and B the total attenuation is 0.80. A linear equation that describes this relation is a + b = 0.80

As Beam 2 passes through grids A and C, the total attenuation is 0.55. The equation that describes this relation is a + c = 0.55. As Beam 3 passes through grid cells B and C, the total attenuation is 0.65 and b + c = 0.65 describes this relation.

$$a + b = 0.80$$
$$a + c = 0.55$$
$$b + c = 0.65$$

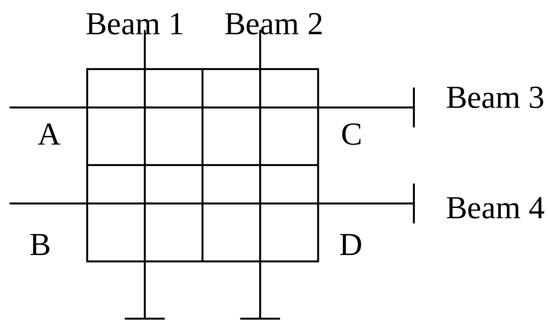
Solving this system, we find that a = 0.35, b = .45, c = 0.2. Comparing these values to the table, one might conclude that cell A contains tumorous tissue, cell B contains bone, and cell C contains healthy tissue.

A. Consider the following data from several patients. Using the three-cell grid above, what can you determine about each patient?

Beam Decrease in Linear Attenuation Units

Patient	Beam 1	Beam 2	Beam 3
W	0.54	0.40	0.52
X	0.65	0.80	0.75
Y	0.51	0.49	0.44
Z	0.44	2.21	2.23

B. Consider the following 4 grid cells.



Observations showed that Beam 1 is decreased by 0.60 units, Beam 2 is decreased by 0.75 units, Beam 3 is decreased by 0.65 units, and Beam 4 is decreased by 0.70 units. Is this enough information to determine the values of a, b, c, and d? Explain your reasoning.

C. Using the following values for a, find b, c, and d for each value of a.

1) a = 0.23
2) a = 0.33
3) a = 0.43

D. Are there sufficient X-rays in to make a determination about the patient? Explain your reasoning.

EXTENSIONS

1. Add two additional X-ray beams in the four- cell grid drawing through the diagonals. Label the diagonal through cells B and C Beam 5. Label the beam through cells A and D Beam 6. Suppose that Beam 5 is decreased by 0.85 units and Beam 6 is decreased by 0.50 units. What can be determined if this information is combined with the information from part B?
2. Can you find an arrangement of exactly 4 beams that will produce a solution?

REFERENCE: Jabon, Nord, Wilson, Coffman and Nord, *Medical Applications of Systems of Linear Equations*, Mathematics Teacher, May, 1996.

PROBLEM 4: BOOK SALE

Each year the County Library has a used book sale. This year the library is selling used books, CDs, and videotapes. All books sell for one price, all CDs sell for one price, and all videotapes sell for one price. Three of your friends found some really good deals. For $25.00 Melissa bought 4 books, 2 CDs, and 2 videotapes. Jeremy bought 3 books, 1 CD, and 2 videotapes for $19.00. Taylor bought 1 book, 4 CDs, and 3 videotapes for $27.00.

Determine the price of each book, CD, and videotape.

"BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

- The FX2.0 has a built-in Tutorial for simultaneous equations that can help users learn how to solve 2-by-2 systems using either substitution or the addition-subtraction menu. From the MAIN MENU, highlight "Tutor" and press [EXE] .
- Highlight "Simul Equation" and press [EXE] .
- Highlight the form you wish to use. If you wish to put in your own values for the constants, press [F2] .
- Type in the desired values, pressing [EXE] after each entry.
- Press [F6] twice and the calculator will set up a tutorial.
- Select either option 1 or 2 as desired.
- Then [F6] and [F1] to move forward and backward through the solution.

This technique can be very powerful in helping students master the skills they need to solve a system of equations. By "busting this barrier," the Algebra FX2.0 can allow the study of higher order, and perhaps, more significant, mathematics.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	6.1, 6.2
AWSM – Focus on Advanced Algebra (1998)	3.1
Glencoe – Algebra 1 (1998)	1.2, 5.6, 8.1, 8.2, 8.3, 8.4
Glencoe – Algebra 2 (1998)	3.1, 3.2, 3.7, 4.6, 4.7
Holt Rinehart Winston – Algebra (1997)	6.1, 6.2, 6.3, 6.4, 6.6, 7.5
Holt Rinehart Winston – Advanced Algebra (1997)	4.3, 4.4, 4.5, 4.6
Key Curriculum – Advanced Algebra Through Data Exploration	9.1, 9.4, 10.1
Merrill – Algebra 1 (1995)	11.2
Merrill – Algebra 2 (1995)	3.1, 3.9, 4.8
McDougal Littell – Algebra 1: Explorations and Applications (1998)	7.2, 7.3, 7.4
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	7.1, 7.4, 7.5
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	9.1
Prentice Hall – Algebra (1998)	5.8, 6.1, 6.2, 6.4
Prentice Hall – Advanced Algebra (1998)	3.6, 4.1, 4.2, 4.5
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	11.1, 11.2, 11.3, 11.7
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	5.2, 5.6
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	11.9, 11.10
Southwestern – Algebra 1: An Integrated Approach (1997)	7.2, 7.3, 7.4, 7.5