CLEMSON ALGEBRA PROJECT
UNIT 6: SYSTEMS OF LINEAR INEQUALITIES

PROBLEM 1: RAISING MONEY

You and your classmates decide to sell sweatshirts and T-shirts to raise money for a school trip. You decide that you should sell at least thirty items, but do not want to exceed 120 items. Based on a small survey of students, you also decide that the number of T-shirts should be at least twice the number of sweatshirts.

A. Assign variables to the unknown quantities and write a system of inequalities that model the given restrictions.

B. Graph the system, indicating an appropriate window and scale and shading the feasible region.

C. Determine the vertices of the polygonal feasible region.

D. Assume the profit on each sweatshirt is $5 and the profit on each T-shirt is $2. What is the maximum profit you can obtain?

E. How many sweatshirts and how many T-shirts should you sell to maximize your profit?

MATERIALS

Casio CFX-9850Ga PLUS or ALGEBRA FX2.0 Graphing Calculator

EXTENSIONS

1. Use inequalities that do not involve “or equal to.”
2. Change the restrictions to ones determined by the class. Use numbers that do not result in integral solutions.
3. Assume different profit margins and determine the maximum profit.

ONE SOLUTION TO PROBLEM 1: RAISING MONEY

A. Assign variables to the unknown quantities and write a system of inequalities that model the given restrictions.

Let x represent the number of sweatshirts and let y represent the number of T-shirts that you intend to sell. Since you want to sell at least thirty items, the sum of x and y must be 30 or greater. Algebraically, we write $x + y \geq 30$.

Similarly, if you do not want to exceed 120 items, the sum of x and y must be 120 or less. Algebraically, this becomes $x + y \leq 120$.

Our third restriction indicated that the number of T-shirts (y) must be greater than or equal to two times the number of sweatshirts. We can write $y \geq 2x$.

If we care to, we can include contextual, common sense inequalities stating that both x and y must be at least (greater than or equal to) 0.

B. Graph the system, indicating an appropriate window and scale and shading the feasible region.

From the MAIN MENU screen, call up the "Graph" menu.

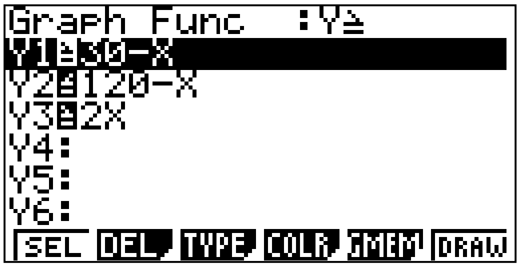
- Delete any functions by pressing [F2] for "Delete" and [F1] to confirm the deletion.

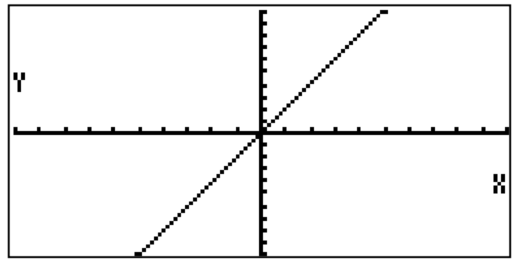
The first inequality we wish to enter is $x + y \geq 30$. First, however, we need to solve for y. Subtracting x from both sides gives us $y \geq 30 - x$.

- To change the TYPE of relation, press [F3], [F6] for more options, and then [F3] to obtain the "greater than or equal to" relationship we are looking for.
- At this point, just type in $30 - x$ and press [EXE].
- To put in the second relation, which is a "less than or equal to" relationship, again solve for y first. Then press [F3] for TYPE, [F6] for more options, [F4] for the "less than or equal to" relation, $120 - x$, and [EXE].

- Again change the type to enter the third relation, $y \geq 2x$. After pressing the up arrow a couple of times so you can see all three relations, your screen should look like the one shown below on the left.

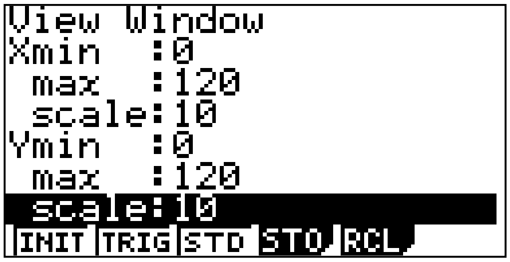
If the standard viewing window is used, you will not see much of the graph. What you do see is shown below right.

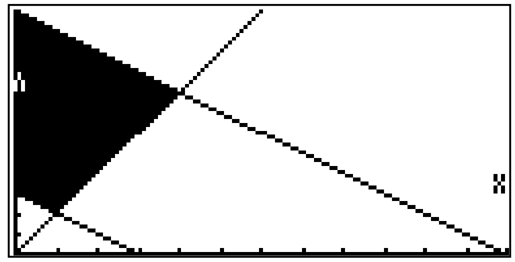




The problem, of course, is with the window. The standard window only shows values between –10 and 10 for both the *x* and *y* variables. To begin a search for a window, we may decide to look at values between 0 and 120 for both *x* and *y*. The logic behind this choice is that we cannot sell a negative number of sweatshirts or T-shirts, and certainly if the total is restricted to 120, neither value alone could exceed 120. A scale of 10 might be appropriate for both axes to avoid an over-abundance of tick marks.

- When looking at the graph, press [SHIFT] [F3] for the viewing window. Type in the desired values, pressing [EXE] after each entry to obtain the screen shown below left.
- Pressing [EXIT] and [F6] will redraw the graph with the new viewing window.
- If you wish, the window may be further refined, perhaps by setting the maximum *x*-value to 50. To do so, press [SHIFT] [F3], make the change and press [EXE], [EXIT], and [F6]. The result is shown below right.





C. Determine the vertices of the polygonal feasible region.

The vertices of the shaded feasible region can be found numerically (by using table values on the calculator if desired), algebraically (by solving for the intersections of pairs of lines), or graphically. An approach that takes advantage of the calculator's capabilities is described below.

Assuming we are restricting ourselves to non-negative values (using $x \geq 0$, the *y*-axis) as the left boundary, our region is framed by a quadrilateral. We wish to find the vertices of this figure. Two of the points are on the *y*-axis; by definition, these are the y-intercepts of two of our boundary lines. Our three lines are $x + y = 30$, $x + y = 120$, and $y = 2x$. Before we find the y-intercepts graphically, let us note that it can be easily determined that the *y*-intercepts are 30, 120, and 0 respectively. Since (0, 0) is not a boundary point, two of the four vertices are (0, 30) and (0, 120).

The calculator also gives us these points quickly. While looking at the graph:

x Press [F5] to access the graph solver

x [F4] for Y-ICPT (the *y*-intercept), the up or down arrow key until the desired line is shown, and [EXE] .

x Repeat this process for each of these three lines. You should quickly identify the two desired points. Again, (0, 0) is not desired.

To find the other two vertices:

x Again access the graph solver by pressing [F5] , but then press [F5] to find the intersection points.

- Use the up or down arrows to find a desired function and press EXE when a function you want is listed.

Note that when you try to find the intersection of Y1 and Y2, you receive a "NOT FOUND" message. This is because the two lines are parallel and have no intersection. Repeat the process for other pairs of lines. The intersection of Y1 and Y3 is (10, 20) and the intersection of Y2 and Y3 is (40, 80). We now have our four vertices. Again, they are (0, 30), (0, 120), (10, 20), and (40, 80).

D. Assume the profit on each sweatshirt is $5 and the profit on each T-shirt is $2. What is the maximum profit you can obtain?

To maximize (or minimize) a value in linear programming, one need only check the vertices of the polygonal feasible region. Since *x* represents the number of sweatshirts and y the number of T-shirts, we need only check the value of the expression $5x + 2y$ at each of the four points found above. Several techniques can be used, but for values such as these, simply plugging into the expression may be the simplest. This is shown below:

(0, 30)	-	5(0) + 2(30) = $60
(0, 120)	-	5(0) + 2(120) = $240
(10, 20)	-	5(10) + 2(20) = $90
(40, 80)	-	5(40) + 2(80) = $360

Clearly, the maximum profit that can be achieved is $360. Students may wish to check other points shaded in the feasible region to evaluate the profit; this may help convince them that maximum and minimum values can occur only at vertices.

E. How many sweatshirts and how many T-shirts should you sell to maximize your profit?

This question has actually been answered within the work for question D above. The point that produced the maximum profit of $360 was (40, 80). Consequently, you should sell 40 sweatshirts and 80 T-shirts.

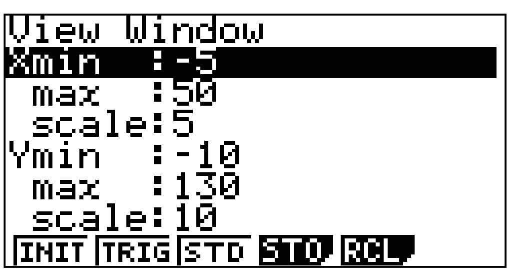
ANOTHER TECHNIQUE WITH LINEAR PROGRAMMING

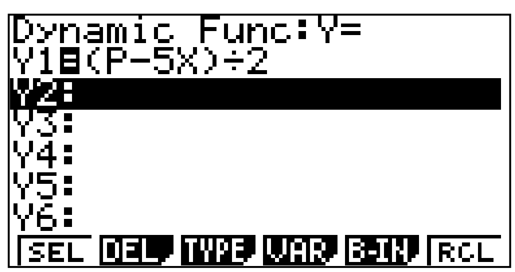
Another technique that can be effective in investigating linear programming problems, such as ***RAISING MONEY***, involves the DYNAMIC capabilities of the calculator. The authors of this material have come to call this technique the "Nina Technique," named after a participant in the pilot sessions who shared her ideas with us.

Begin as normal, using the GRAPH menu to establish the feasible region. The window for this problem has been changed to the one shown below left. Then,

- With the graph displayed, press OPTN , F1 for picture, F1 to Store the picture, and F1 or any other function key to store the graph as a picture in any desired memory location.
- Press MENU for the MAIN MENU. Call up the "Dynamic Function" screen.
- Press SHIFT MENU for SET UP, scroll down to "Background," press F2 for Picture, and F1 or whatever function key is needed for the location in which you stored the picture. Then press EXE to return to the "Dynamic Function" screen. (Remember to set the Background to None later.)
- Highlight each of the functions and delete them by pressing F2 and F1 .

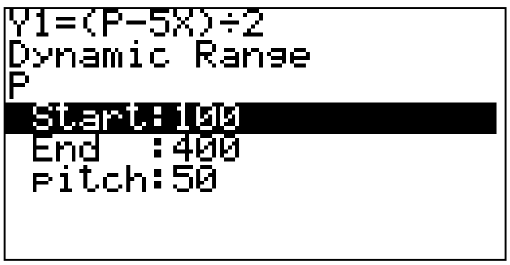
Our profit function can be represented algebraically as $P = 5x + 2y$. We want to see what the maximum profit is that remains within our feasible region. Solving this function for y, we have $y = \dfrac{P - 5x}{2}$. Enter this function as shown below right.

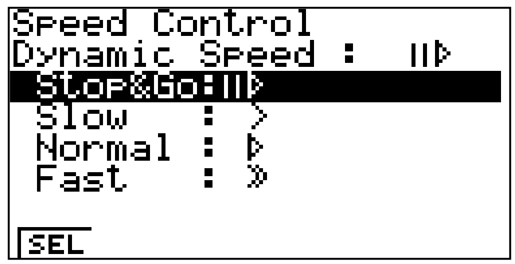




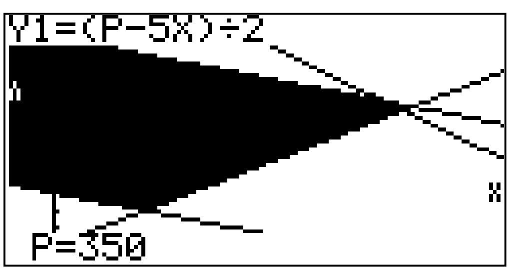
We now wish to set P up as a dynamic variable, that is one that can vary according to conditions we set. To do so:

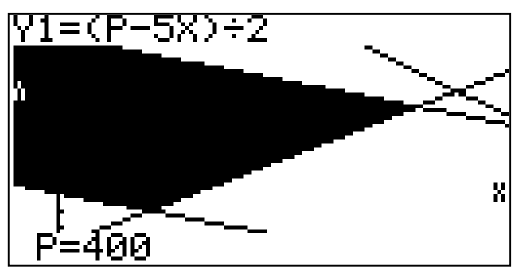
- Press F4 for Variable. P will be automatically listed as the dynamic variable.
- Press F2 for Range. Select values that you think might describe the possible profits. If you are unhappy with the values you have chosen, you can simply try again. One possible set is shown below left. Press EXE after entering each value.
- When you are finished, press EXIT .
- To select the speed, press F3 . Highlight the speed you want and press F1 to select it. “Stop&Go” has been selected, as shown below right.





- Press EXIT to return to the primary “Dynamic” screen and then F6 . This will take your calculator a few moments to set up. If you get a MEMORY error, try setting a different range of values for P.
- Press EXE to move the profit function through the feasible region. You may notice that when P equals $350, the profit line is still within the feasible region (shown below left), but when P is $400, the profit line has left the feasible region (shown below right).





Changing the range so that the pitch is smaller allows you to be more accurate. This technique can provide an excellent visual for the students.

PROBLEM 2: THE SNACK PROBLEM

Assume you like snacks and insist on having at least one serving of dry roasted peanuts and one serving of potato chips each day. Each serving of the peanuts contains 15% of the recommended daily allowance of saturated fat; each serving of potato chips contains 10%. Each serving of the peanuts contains 12% of the recommended amount of dietary fiber; each serving of potato chips contains 5%. You determine that you want to consume no more than 60% of the recommended allowance of dietary fat from these two snacks, but you want to get at least 30% of the recommended allowance of fiber from them.

A. Sketch the feasible region relating the number of servings of potato chips you might have with the number of servings of dry roasted peanuts.

B. The peanuts cost 12.4 cents per serving and the potato chips cost 23.2 cents per serving. If you keep within the constraints mentioned, how many servings of each should you have each day to minimize your cost?

ONE SOLUTION TO PROBLEM 2: THE SNACK PROBLEM

First we need to identify our variables.

- Let x represent the number of servings of dry roasted peanuts we will have per day.
- Let y represent the number of servings of potato chips we will have per day.

Next we need to write our constraints mathematically. First we want both x and y to be at least one. Second, we want the amount of saturated fat to total no more than 60% of the recommended allowance. Finally, we want the amount of dietary fiber to be at least 30%. Mathematically we can write:

1) $x \geq 1$

2) $y \geq 1$

3) $0.15x + 0.10y \leq 0.60$

4) $0.12x + 0.05y \geq 0.30$

To address part A, we will graph the system. We will use the graphing window to set minimum values for x at 1. We will enter the second inequality directly in the calculator. For the third and fourth inequalities, we need to first solve for y. We could use the “Algebra” mode on the ALGEBRA FX2.0 for assistance, but we would hope this is not necessary, especially if we are not concerned with simplification. Solving inequalities 3) and 4) for y, we obtain:

3) $y \leq (0.60 - 0.15x) \div 0.10$

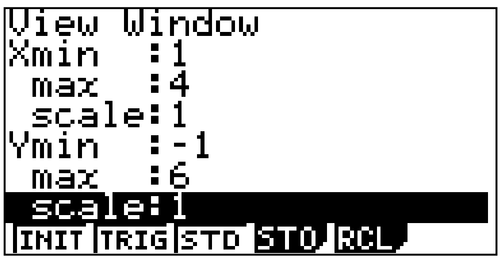
4) $y \geq (0.30 - 0.12x) \div 0.05$

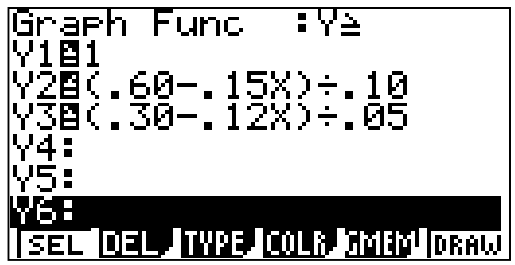
We will now set the window. Again, for the first inequality, we set the minimum x value at 1. Although we could set the minimum y value at 1 also because of the second inequality, we will want to see the bottom of the graph clearly. Consequently, we will set it at –1. Inequality 3) shows that the biggest x and y can be are 4 and 6, respectively. From the GRAPH menu, press [SHIFT] [F3] and enter the values, pressing [EXE] after each entry. Your screen should look like the one shown below left. To return to the primary “Graph” screen, press [EXIT] .

To enter inequalities 2), 3) and 4), from the primary “Graph” screen, delete or de-select and relations that have already been input. Then:

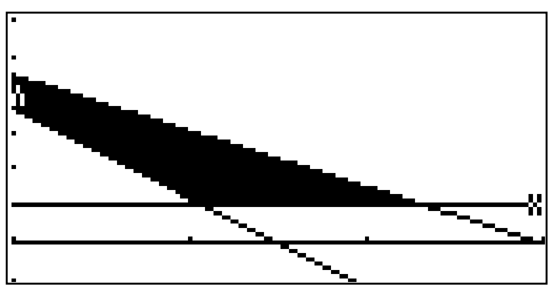
- x Press F3 to change the type.
- x Press F6 for more options.
- x Press F3 for less than or equal to.

Type in inequality 2) and press EXE . Repeat this process to enter inequalities 3) and 4), but press F3 when you need greater than or equal to. After typing in the three inequalities (and using the arrow keys to view them simultaneously), your screen should look like the one shown below right.





To view the feasible region, press F6 . The result is shown below.



To address question B of this problem, we need to find the coordinates of the vertices of our polygonal region. Since we intend to minimize the cost, we need only look at the vertices that are left and below. However, for students who have not yet made this connection, you may want to find all four vertices. Pressing F1 will give us the trace feature of the calculator. When we first do this, we obtain the point (1, 1), which is not a vertex of the feasible region. Pressing the down arrow gives us the point (1, 4.5), the upper left vertex. Pressing the down arrow again gives us the other vertex on the left, the second of the two in which we are interested. This is the point (1, 3.6).

To find the other two vertices, we can once again take advantage of the calculator.

- Press F5 to access the graph solver.
- Press F5 for “Intersection.”
- Press EXE to accept Y1 as one of the relations we’re interested in.
- Press EXE to accept Y2 as the other.

The cursor will move to our intersection point, (3.333, 1). This is the farthest right vertex of our feasible region. To find the fourth point,

- Press F5 to access the graph solver.
- Press F5 for “Intersection.”
- Press EXE to accept Y1 as one of the relations we’re interested in.
- Press the down arrow to show Y3 as the other.
- Press EXE to accept Y3.

The cursor will move to our final vertex, which is at (2.083, 1). All that is left to do is to determine how much each will cost, and select the one that yields the minimum. Since each serving of peanuts costs 12.4 cents and each serving of potato chips costs 23.2 cents, our total cost is determined by computing $12.4x + 23.2y$. Plugging in our values we obtain the following:

$$(1,\ 4.5) \rightarrow 12.4 \times 1 + 23.2 \times 4.5 = 116.8$$

$$(1,\ 3.6) \rightarrow 12.4 \times 1 + 23.2 \times 3.6 = 95.92$$

$$(3.333,\ 1) \rightarrow 12.4 \times 3.333 + 23.2 \times 1 = 64.53$$

$$(2.083,\ 1) \rightarrow 12.4 \times 2.083 + 23.2 \times 1 = 49.03$$

Note that the last point gives us the minimum cost. Interpreting this, we find that to meet our criteria, we should have just over two servings of peanuts and exactly one serving of potato chips each day. Our cost for doing this is just over 49 cents.

PROBLEM 3: CARS AND TRUCKS

A car dealership keeps cars and trucks on its lot. It has room for no more than 500 vehicles. Cars cost the dealer an average of $20,000 each, and trucks cost the dealer an average of $12,000 each. To keep the dealership viable, the owner decides to spend at least $7,760,000 purchasing vehicles from the manufacturer. To keep within the budget, the dealer also decides to spend no more than $8,800,000 on purchasing vehicles.

A. Sketch the feasible region relating the number of trucks at the dealership to the number of cars.

B. Suppose that the dealer makes, on average, a profit of $2,000 for each car and $1,500 for each truck. Assuming all cars and trucks will be sold, how many of each type should the dealer purchase? How much profit will be realized?

PROBLEM 4: BALANCING YOUR WORKLOAD

To pay for car insurance and other expenses, you need to work during the school year. Although you would prefer not to go to your job Monday through Thursday, your employer insists that you work at least four hours total during those days to keep your job, which pays $6 per hour. Your parents are concerned about your grades and insist that you must study at least twice as many hours as you work on Monday through Thursday. You figure you have no more than 24 hours on Monday through Thursday to devote to a combination of work and study.

A. Sketch the feasible region, relating the hours you spend studying on Monday through Thursday with the hours you spend working.

B. Suppose as an incentive and to help you make ends meet, your parents pay you $2 for every hour that you study on Monday through Thursday. How many hours should you work and how many hours should you study to maximize your income from these four days. How much income would you receive?

"BUSTING BARRIERS" WITH THE ALGEBRA FX 2.0

Although it should not be necessary for this problem, the "Algebra" menu on the ALGEBRA FX2.0 can be used to help students solve equations for *y*.

- From the MAIN MENU, highlight "Algebra" and press [EXE] .
- Press [F1] for the TRNS menu.
- Press the appropriate number for "Solve."
- Type in the equation you wish to solve, a comma, the variable you wish to solve for, close the parentheses, and press [EXE] . The top of the screen might look like this: solve$(X+Y=30,Y)$. NOTE: Although the FX2.0 can solve literal equations, it cannot solve literal inequalities. This is to avoid division when the sign of the divisor is unknown. To solve literal inequalities, simply use the FX2.0 to solve the literal equation and substitute the appropriate sign.

Using the Algebra Menu to Solve Manually

In addition to solving inequalities for specific variables, the FX 2.0 can also assist students in learning the process for themselves. From the MAIN MENU, access the "Algebra" menu and follow the steps below.

- To clear other entries, press the function key for "Clear," the number for "ALL EQUATIONS," and [EXE] for yes.
- At the cursor, type in the equation or inequality you want to work on and press [EXE] . To obtain a greater than or less than sign, press the function key for "Equation," the number for "Inequalities," and the number of the desired sign. For example you might type in $5x+2y\geq 100$. This inequality is labeled as equation 1.
- Suppose we wanted to solve this inequality for *y*. All we need do to begin is simply tell the calculator to subtract 5*x*. The calculator assumes you are referring to the equation in its memory. Press [EXE] . This becomes equation 2, and should appear as $5x+2y-5x\geq 100-5x$.

- To simplify equation 2, press [F1] for the TRNS menu, the number for simplify, and [EXE] . The result has isolated the Y term, showing us that $2y \geq -5x + 100$. This is labeled as equation 3. One of the many exciting features of the calculator is that, even if the student tells the calculator to do something wrong, the calculator does it, but the student can recognize that it does not help. The student can then return to the previous equation and try something else.
- The next step, of course, is to divide by 2. Simply press the division sign, 2, and press [EXE] . The calculator shows $\frac{2y}{2} \geq \frac{-5x + 100}{2}$.
- This needs to be simplified, so again press [F1] for the TRNS menu, the number for simplify, and [EXE] . The calculator shows that $y \geq \frac{-5x}{2} + 50$, using true fraction notation.

This technique can be a very effective tool in helping students master the skills they need to solve inequalities for particular variables. By “busting this barrier” that impedes the progress of so many students, the ALGEBRA FX 2.0 can then allow the study of higher order and, perhaps, more significant mathematics.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	6.3
AWSM – Focus on Advanced Algebra (1998)	3.2
Glencoe – Algebra 1 (1998)	8.5
Glencoe – Algebra 2 (1998)	3.4, 3.5, 3.6
Holt Rinehart Winston – Algebra (1997)	
Holt Rinehart Winston – Advanced Algebra (1997)	4.8, 4.9
Key Curriculum – Advanced Algebra Through Data Exploration	9.5, 9.6, 9.7
Merrill – Algebra 1 (1995)	11.6
Merrill – Algebra 2 (1995)	3.4, 3.6, 3.7
McDougal Littell – Algebra 1: Explorations and Applications (1998)	7.6
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	7.6, 7.7
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	10.7
Prentice Hall – Algebra (1998)	6.6, 6.7
Prentice Hall – Advanced Algebra (1998)	4.3
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	11.8
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	5.8, 5.9, 5.10
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	
Southwestern – Algebra 1: An Integrated Approach (1997)	8.2, 8.3, 8.4