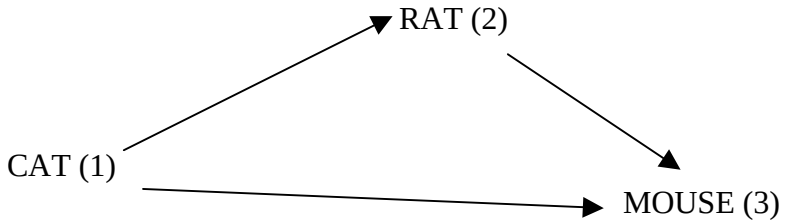
CLEMSON ALGEBRA PROJECT
UNIT 8: MATRICES

PROBLEM 1: FOOD WEB OF SELECTED ANIMALS

The line diagram below is called a digraph and represents a small food web. The directed segment joining cat and rat, for example, indicates that cats eat rats. Numbers 1, 2, and 3 are associated with each animal for later reference.



The relationship expressed in this digraph can be represented by a matrix if we let the numbers 1, 2, and 3 represent the respective rows and columns in a 3-by-3 matrix. In constructing the matrix to convey the information of the digraph, let the position for each entry be designated by the ordered pair (*i, j*), the *i* indicating the row position and the *j* indicating the column position. Put a 1 in that position if *i* eats *j*, and put a 0 in that position if *i* does not each *j*. Thus the matrix associated with the above digraph, which we will call F, is $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. The three 1's in the matrix represent the three directed segments of the digraph.

The matrix F^2 represents indirect food sources. For example, in row 1 column 3 of F^2, a "1" appears because cats use mice as an indirect food source; that is cats eat rats, and rats eat mice. Determining why this happens is a worthwhile exploration in itself.

Now suppose that because of damp, rainy weather, the insect population of an area has increased dramatically. The insects are annoying to people and animals. State authorities are in favor of using an insecticide that would literally wipe out the entire insect population. You, as an employee of the Environmental Protection Agency, must determine whether this action will be detrimental to the environment.

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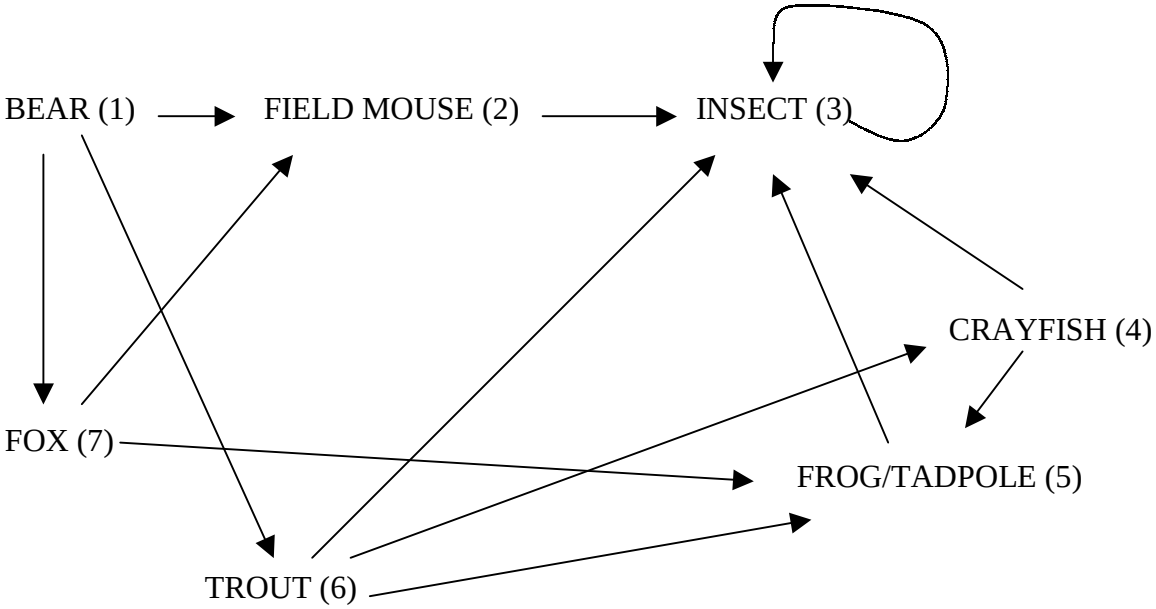
Consider the following possible digraph of a food web for seven animals, including the insects that are causing the problem.



PART 1

A. Construct the associated matrix to represent this web. Call it Matrix A. Notice that rows 2, 3, and 5 each contain a single 1. What does this indicate?

B. Column 1 contains all 0's. What does a column of all 0's indicate?

C. Notice that bears and trout have the most direct sources of food. This can be determined by finding the sum of the numbers in the rows. Find the sums of the seven rows.

D. Column 3 has the most 1's. What does this suggest about the food web?

E. The matrix A^2 denotes indirect (through one intermediary) sources of food. Find A^2. Notice that column 3 contains all nonzero numbers. What does this indicate?

F. Find $A + A^2$ and the associated row sums. This matrix denotes the total number of direct and indirect food sources of food for each animal.

G. What animal has the most food sources?

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PART 2

Now suppose that an insecticide has been introduced and all of the insects have been killed. Several animals will lose a food source.

H. Construct a new Matrix B to represent the food web with no insects. What effect does this have on the overall animal population?

I. What has happened to the row sums? What has happened to the food source of the tadpoles and the field mice? What are the implications?

J. Find B^2 and $B + B^2$.

K. What are the row sums? Compare these answers with those of the original Matrix A.

L. Will all the animals be affected by the insecticide? Which animals will be least affected?

M. Organize and summarize your findings in a brief report to convince the authorities that this insecticide is harmful to the total environment.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

REFERENCE: Mathematical Modeling in the Secondary School Curriculum, A Resource Guide of Classroom Exercises, Edited by Frank Swetz and, J.S. Hartzler, NCTM, 1991.

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ONE SOLUTION TO PROBLEM 1: FOOD WEB OF SELECTED ANIMALS

PART 1

A. Construct the associated matrix to represent this web. Notice that rows 2, 3, and 5 each contain a single 1. What does this indicate?

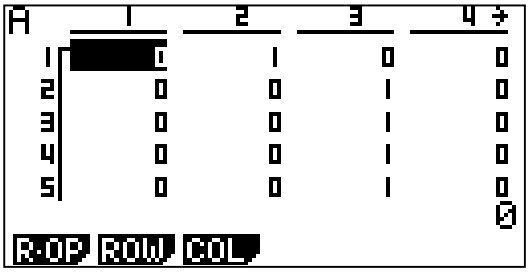
We will use the graphing calculator for our matrix work. From the MAIN MENU, call up "Matrices." Then

- Define the dimensions of Matrix A. Type 7, press [EXE] , again type 7, and again press [EXE] .
- Type in each number in the matrix, reading across the row and pressing [EXE] after each entry. Press [EXIT] when finished.

The entire matrix will not show on one screen, but it should be equivalent to the matrix shown below.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

On your calculator, the first five rows of Matrix A should look as below.



The single 1 in rows 2, 3, and 5 suggests that, at least for the animals listed in the food web, field mice, insects, and frogs/tadpoles have only one source of food.

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B. Column 1 contains all 0's. What does a column of all 0's indicate?

A column of 0's indicates that the animal has no predator. Here it tells us that none of the listed animals eats bears.

C. Notice that bears and trout have the most direct sources of food. This can be determined by finding the sum of the numbers in the rows. Find the seven row sums.

Row sums in order are {3, 1, 1, 2, 1, 3, 2}.

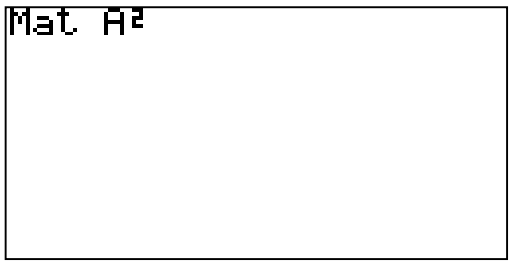
D. Column 3 has the most 1's. What does this suggest about the food web?

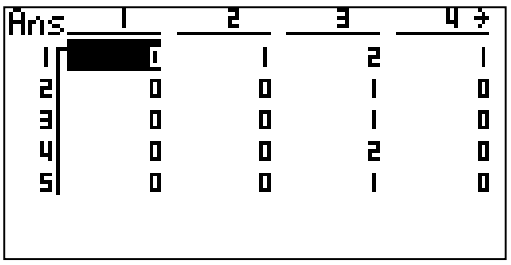
More animals rely on insects for food than any other source.

E. The matrix A^2 denotes indirect (through one intermediary) sources of food. Find A^2. Notice that column 3 contains all nonzero numbers. What does this indicate?

From the MAIN MENU, select "Run." Then,

- Press [OPTN] , [F2] for "Matrices," [F1] for matrix names, [ALPHA] for letters, the key with the letter A on it (the same key that is used for the *x*-variable), and the squaring key. Your screen should look like the one below left.
- Press [EXE] to obtain the result. See below right.





The fact that column 3 does not contain any 0's indicates that every animal eats an animal which feeds on insects. In other words, insects are an indirect source of food for every animal listed in the web.

The entire matrix follows.

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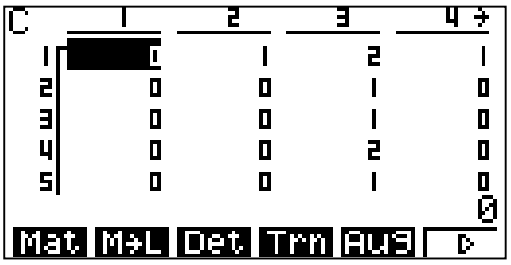
$$\begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

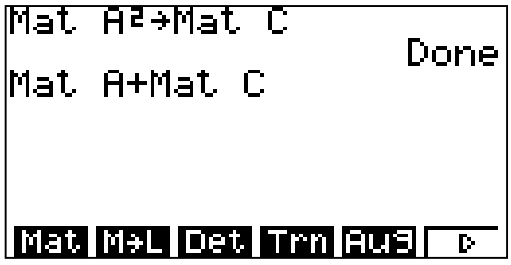
F. Find $A + A^2$ and the associated row sums. This matrix denotes the total number of direct and indirect food sources of food for each animal.

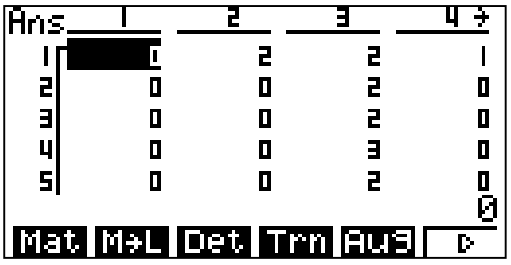
First, we want to store A^2 in for Matrix C (we have already identified a purpose for Matrix B.)

- To do so, after pressing the [EXE] key above, press [AC/ON] to clear the screen.
- Now repeat the procedure for squaring Matrix A, then press the store key (the right arrow), [F1] for matrix names, [ALPHA] and the key with C on it. Before you press [EXE] , your screen should like the one below left.
- After you press [EXE] your screen should look like the one below right.



Had we chosen to do so, we could have done this when we first squared A. We now wish to add Matrix A with Matrix C. Simply using [F1] to tell the calculator you are working with matrices, perform the operation as shown below left. After pressing [EXE] your screen should look like the one below right.





The entire matrix sum is shown below.

$$\begin{bmatrix} 0 & 2 & 2 & 1 & 2 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 & 0 \end{bmatrix}$$

G. What animal has the most food sources?

The bears have the most food sources. The sum of the first row is 9, the highest of all the row totals.

PART 2

H. Construct a new matrix B to represent the food web with no insects. What effect does this have on the overall animal population?

From the MAIN MENU, choose "Matrix." Highlight B and again make it a 7 by 7 matrix (so we can keep the same labels), but construct the matrix as if there were no arrows going into or out of the insects. Obviously, there will be a large impact upon these animals. Your matrix should be equivalent to the following:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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I. What has happened to the row sums? What has happened to the food source of the tadpoles and the field mice? What are the implications?

The row sums of the animals that depend on insects have decreased by 1. The only food source for the tadpoles and the field mice has disappeared, and they will soon die.

J. Find B^2 and $B + B^2$.

Using the same technique as described earlier, we find that B^2 and $B + B^2$ are:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

K. What are the row sums? Compare these answers with those of the original matrix A.

The row sums of $B + B^2$ are {7, 0, 0, 1, 0, 3, 2}, considerably smaller than before the insects were eliminated.

L. Will all the animals be affected by the insecticide? Which animals will be least affected?

All the animals will be affected by the insecticide. The bear will be affected the least.

M. Organize and summarize your findings in a brief report to convince the authorities that this insecticide is harmful to the total environment.

Answers will vary, but all should recognize the potential impact of eliminating an animal from the food web, especially one low on the food chain.

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PROBLEM 2: THE DOG WALKERS

Max and Moritz will be going off to college in a few years. They have been attending Junior Achievement meetings and decide they would like to start their own business to earn money for college. Because they only have $200.00 start up capital, Max and Moritz decide to start a business that offers a service.

After several weeks of observing their neighborhood, they decide that a dog walking service is needed. In addition to establishing their own web page, Max and Moritz have 200 advertisements for the M & M Dog Walkers to send to families in their neighborhood. To get the ads out in a timely manner, Max and Moritz hire three people to fold the ads, stuff and seal the envelopes, and apply the address labels they printed on their computers. They will pay $0.03 to fold an advertisement, $0.06 to stuff and seal an envelope, and $0.02 to apply an address label.

Person 1 folds 58 advertisements, stuffs and seals 60 envelopes, and applies 50 address labels.

Person 2 folds 77 advertisements, stuffs and seals 72 envelopes, and applies 78 address labels.

Person 3 folds 65 advertisements, stuffs and seals 68 envelopes, and applies 72 address labels.

A. Create a table showing the amount of work each person does.
B. Determine how much each person will be paid. Explain how you arrived at your answer.
C. Write a matrix representing the amount paid for each action. Label the rows and columns.
D. Write a matrix representing the number of actions performed by each person. Label the rows and columns.
E. Use matrix multiplication to determine how much each person will be paid. How does this answer compare with your answer from problem B?
F. How would you explain matrix multiplication to your friend who was absent from school when you completed this activity?

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ONE SOLUTION TO PROBLEM 2: THE DOG WALKERS

A. Create a table showing the amount of work each person does.

NOTE: The rows and columns in the table below may be transposed.

ACTION	PERSON 1	PERSON 2	PERSON 3
Fold	58	77	65
Stuff / Seal	60	72	68
Label	50	78	72

B. Determine how much each person will be paid. Explain how you arrived at your answer.

Simply multiply the amount paid for each action by the number of each action and then sum for each person.

Person 1 will be paid \$6.34 $(58*.03+60*.06+50*.02)$.

Person 2 will be paid \$8.19 $(77*.03+72*.06+78*.02)$.

Person 3 will be paid \$7.47 $(65*.03+68*.06+72*.02)$.

C. Write a matrix representing the number of actions performed by each person. Label the rows and columns.

Again, rows and columns could be transposed as shown below. The two possible matrices have been labeled as ***C1*** and ***C2***.

C1	**P1**	**P2**	**P3**
Fold	58	77	65
Stuff/Seal	60	72	68
Label	50	78	72

C2	**Fold**	**Stuff/Seal**	**Label**
P1	58	60	50
P2	77	72	78
P3	65	68	72

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D. Write a matrix representing the amount paid for each action. Label the rows and columns.

The two possibilities, a one-by-three and a three-by-one, are shown below. They have been named ***D1*** and ***D2***.

D1	**Fold**	**Stuff/Seal**	**Label**
Amount	0.03	0.06	0.02

D2	**Amount**
Fold	0.03
Stuff/Seal	0.06
Label	0.02

E. Use matrix multiplication to determine how much each person will be paid. How does this answer compare with your answer from problem B?

If we multiply ***D1*** by ***C1*** we will use the row in ***D1***, the prices for each of the items, and multiply by it the numbers of items each person completed, as listed in the columns of ***C1***. Alternately, we could multiply ***C2*** by ***D2***. Because matrix multiplication is not commutative, students may have difficulty setting up the multiplication. We should, however, obtain the same answers as we did in problem B.

- From the MAIN MENU, choose "Matrix."
- Make Matrix C a three-by-three matrix and enter matrix ***C1*** shown above, pressing [EXE] after each entry. Press [EXIT] when finished.
- Make matrix D a one-by-three matrix as shown in ***D1***, again pressing [EXE] after each entry. Press [EXIT] when finished.
- Press [MENU] and choose "Run" from the MAIN MENU. Also press [OPTN] and [F2] for matrix options to be available.
- Using [F1] before the matrix name, tell the calculator to calculate Matrix D times Matrix C. Alternatively, tell the calculator to multiply the transpose of Matrix C (use [F4] for the transpose) times the transpose of Matrix D. See below left for the two possible ways to enter this.

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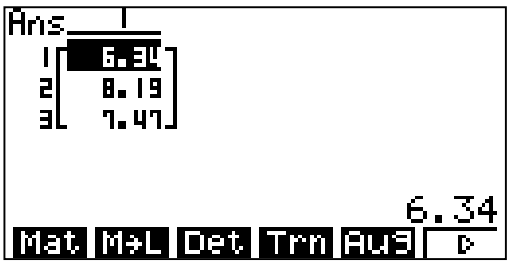
- Press EXE with whatever operation you have chosen. See below right.



The answers should be the same as those we obtained in problem B.

F. How would you explain matrix multiplication to your friend who was absent from school when you completed this activity?

Answer will vary. What is important for students to work on is matching the rows in the left matrix with the columns in the right matrix.

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PROBLEM 3: INVESTMENT PLANS

In order to make economics more relevant to students, teachers often have bankers and stockbrokers speak to their classes. One such investment counselor suggested that students should invest their funds in several different mutual funds. These funds allow the investor to put her/his money in a combination of low risk stocks, medium risk stocks, and high risk stocks. She explained three investment plans to the students.

Plan A:	Low Risk Stocks	50%
	Medium Risk Stocks	35%
	High Risk Stocks	15%
Plan B:	Low Risk Stocks	65%
	Medium Risk Stock	25%
	High Risk Stock	10%
Plan C:	Low Risk Stock	75%
	Medium Risk Stock	20%
	High Risk Stock	5%

Suppose you have saved $4500.00 and you want to try and increase your college fund by investing in all three plans. Your parents agree to allow you to invest in all three plans, if you invest most of your money in the most conservative plan. If you invest $1000.00 in Plan A, $1500.00 in Plan B and $2000.00 in Plan C, how much money will you have invested in low risk stocks, medium risk stocks and high risk stocks? Explain how matrix multiplication can be used to obtain the solution.

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PROBLEM 4: PLANNING A ROUTE

Elaine is a graduate student at Clemson University, which is located in Clemson, SC. She lives in Simpsonville and commutes to Clemson each day. Because of construction, she is looking for alternate routes to Clemson. After examining the map, she discovers that there are several alternate routes. Towns along the various routes include Piedmont, Pelzer, Mauldin, Greenville, and Easley.

A. Create a determination matrix to display the map information. Use the town names to label the rows and columns. If there is a direct route between two towns, enter a 1 in the appropriate cell. If not, enter a 0.

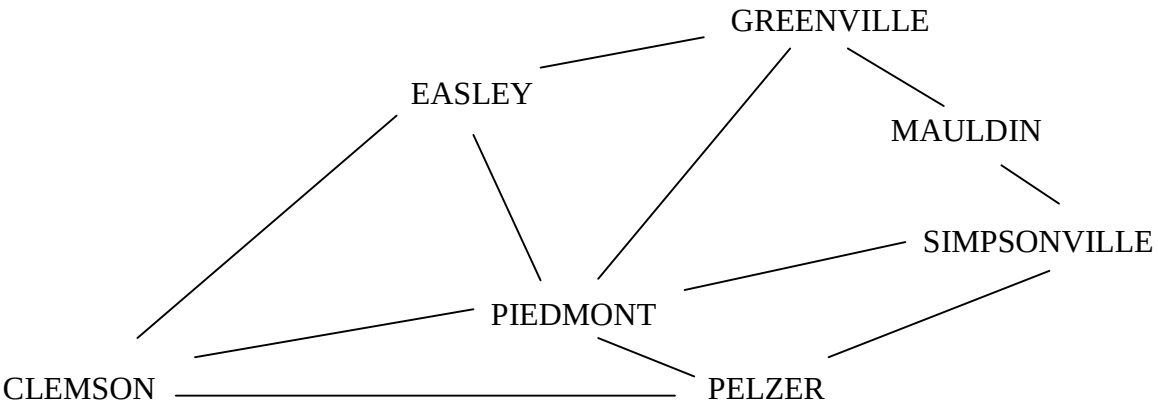
B. In how many ways can Elaine get to Clemson without going through any other town? Explain your reasoning in terms of your determination matrix.

C. Use matrix operations to determine the number of ways Elaine can get to Clemson by going through only one other town. Explain your reasoning.

D. What do the zero entries in the new matrix represent?

E. Use matrix operations to determine the number of ways Elaine can get to Clemson by going through two other towns. Explain your reasoning.

F. How many 2-town routes are there among all seven towns? Explain.



EXTENSIONS

1. Use a map of your region to create and explore a determination matrix.
2. Use a flight schedule to create determination matrices.

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TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	1.1, 6.2
AWSM – Focus on Advanced Algebra (1998)	3.1
Glencoe – Algebra 1 (1998)	2.3
Glencoe – Algebra 2 (1998)	4.1, 4.2, 4.3, 4.6, 4.7
Holt Rinehart Winston – Algebra (1997)	7.1, 7.2, 7.3, 7.4, 7.5
Holt Rinehart Winston – Advanced Algebra (1997)	4.1, 4.2, 4.4, 4.5, 4.6, 4.9
Key Curriculum – Advanced Algebra Through Data Exploration	8.6, 8.7, 9.4
Merrill – Algebra 1 (1995)	
Merrill – Algebra 2 (1995)	4.4, 4.5, 4.6, 4.8
McDougal Littell – Algebra 1: Explorations and Applications (1998)	1.8, 12.2
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	2.4
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	
Prentice Hall – Algebra (1998)	1.8
Prentice Hall – Advanced Algebra (1998)	3.1, 3.2, 3.3, 3.5, 3.6, 4.6
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	4.1, 4.2, 4.3, 5.5, 5.6
Southwestern – Algebra 1: An Integrated Approach (1997)	1.6, 7.5

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