CLEMSON ALGEBRA PROJECT
UNIT 9: POLYNOMIALS

PROBLEM 1: CONES FOR SHAVED ICE

The Parent Teacher Student Association would like to sell cones of flavored shaved ice at each football game to raise money for a new computer lab. The Fund Raising Committee has searched for paper cones to use, but the cones they have found are either too large or too small. The PTSA asks the Math Team to design a conical cup from a circle that has a diameter of 17 centimeters and has the greatest volume.

A. Recall that a cone is made from a circle by removing a sector of arc length x. Cut out a circle and remove a sector from the circle and build a cone.

B. What is the slant height of the cone?

C. Without measuring, determine the radius of the base of the cone you have made in terms of the arc length that you removed from the circle.

D. Without measuring, determine the height of the cone in terms of the arc length you removed from the circle.

E. Write a formula for the volume of the cone in terms of the arc length that you removed.

F. Determine the arc length that must be removed that will result in the maximum volume of the cone.

G. Determine the dimensions of the cone. Make a cone with these dimensions.

H. Could you easily manufacture a cone with these dimensions? Explain your reasoning.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

Scissors

Paper

Tape

ONE SOLUTION TO PROBLEM 1: CONES FOR SHAVED ICE

A. Recall that a cone is made from a circle by removing a sector of arc length *x*. Cut out a circle and remove a sector from the circle and build a cone.

A template may be best for drawing and then cutting out the circle. As the problem suggests, make sure the circle has a radius of 8.5 centimeters.

B. What is the slant height of the cone?

The slant height of the cone is the radius of the original circle; therefore the slant height is 8.5 cm.

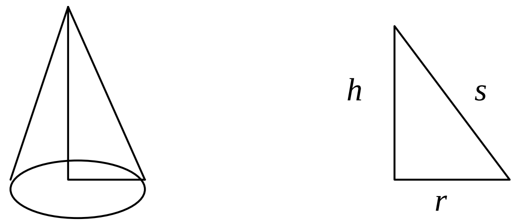
C. Without measuring, determine the radius of the base of the cone you have made in terms of the arc length that you removed from the circle.

The circumference of the base of the cone is the circumference of the original circle minus the arc length that was removed. If we let R represent the radius of the original circle from which the cone was made and x represent the length of the arc that was removed, the circumference of the base is $2\pi R - x$. Since we know that R is 8.5 centimeters, the circumference is $2\pi * 8.5 - x = 17\pi - x$ centimeters.

If we now let r represent the radius of the base, then $2\pi r = 17\pi - x$. Solving for r, we find that

$$r = \frac{17\pi - x}{2\pi}.$$

D. Without measuring, determine the height of the cone in terms of the arc length you removed from the circle.



Let x = the length of the arc removed from the original circle

Let h = the height of the cone

Let r = the radius of the base of the cone, which is $r = \dfrac{17\pi - x}{2\pi}$

Let s = the slant height of the cone, which is 8.5.

Using the Pythagorean Theorem,

$$s^2 = r^2 + h^2$$

$$8.5^2 = \left(\frac{17\pi - x}{2\pi}\right)^2 + h^2$$

$$8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2 = h^2$$

$$h = \sqrt{8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2}$$

E. Write a formula for the volume of the cone in terms of the arc length that you removed.

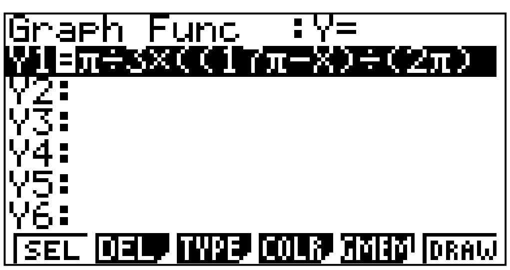
The general form for the volume of a cone is $V = \dfrac{\pi}{3} r^2 h$. Substituting our values, we obtain the following:

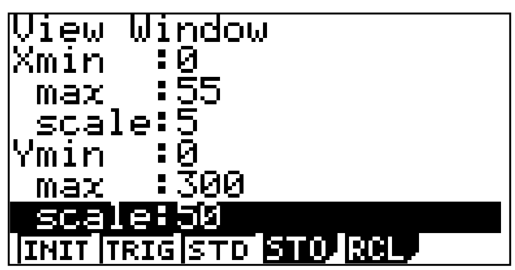
$$V = \frac{\pi}{3}\left(\frac{17\pi - x}{2\pi}\right)^2 \sqrt{8.5^2 - \left(\frac{17\pi - x}{2\pi}\right)^2}$$

F. Determine the arc length that must be removed that will result in the maximum volume of the cone.

We will solve this with a graph. From the MAIN MENU, choose “Graph.”

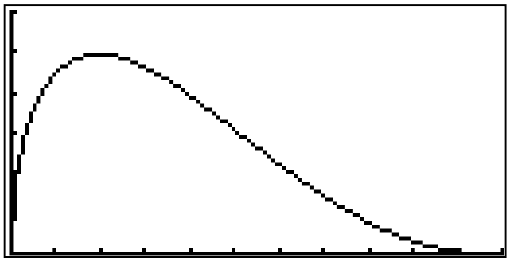
- Delete any functions that are there by highlighting them and pressing [F2] followed by [F1] . Alternatively, de-select them (turn them off) by highlighting them and pressing [F1] .
- Type the formula for the volume of the cone into Y1 and press [EXE] . This is fairly complicated. Be careful with the use of parentheses. The keys are: $\pi \div 3 \times ((1\ 7\ \pi - x) \div (2\ \pi))^2 \times \sqrt{} (8.5^2 - ((17\ \pi - x) \div (2\ \pi))^2)$. See below left for the beginning of this.
- Next, we will set the viewing window. Press [SHIFT] [F3] to obtain the viewing window. Values of x less than 0 have no meaning in this problem, so set the minimum x-value at 0. If x should be greater than 17π, the volume would be negative, so restrict x to values less than 55. Negative values for the volume are also meaningless; therefore set the minimum y-value at 0. Assume that the volume will be no greater than 300 cubic centimeters. Remember to press [EXE] after each entry. See below right for a possible window.

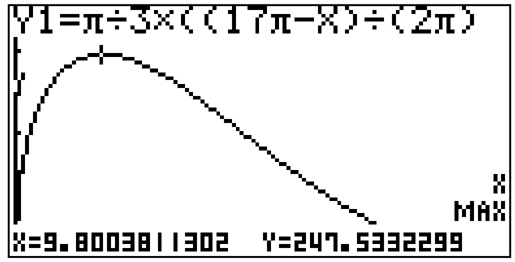




- When the window has been set, press [EXIT] and then [F6] to draw the graph. See below left.
- To determine the value of x that produces the greatest volume, press [F5] for the graph solver followed by [F2] for maximum. The calculator shows us

that if we cut an arc length of 9.8 centimeters, we will achieve the maximum volume, which is 247.5 cubic centimeters. See below right.

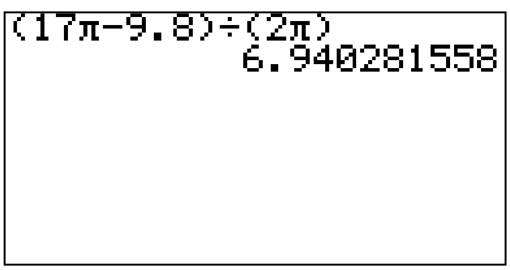


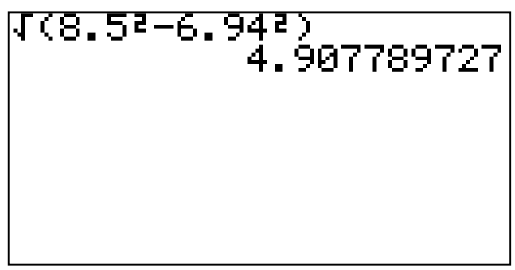


G. Determine the dimensions of the cone. Make a cone with these dimensions.

We know that $r = \dfrac{17\pi - x}{2\pi}$. To find the radius of the cone, we need to substitute 9.8 for x into the formula. From the MAIN MENU, choose “Run.”

- Type in the formula as shown below left and press EXE . The radius is 6.940 centimeters.
- To find the height of the cone, we can substitute 8.5 for s and 6.94 for r into the formula $s^2 = r^2 + h^2$. Since $h = \sqrt{s^2 - r^2}$, we need to find the value of $\sqrt{8.5^2 - 6.94^2}$. See below right to see how we obtained a height of 4.9 cm.





H. Could you easily manufacture a cone with these dimensions? Explain your reasoning.

Since there is no such thing as a perfect measure, the quality of our machinery would determine the precision to which we can build the cones. Note that the radius is approximately twice the height. Consequently, these dimensions may not be optimal if a scoop of shaved ice is to be placed on top the ice in the cup.

PROBLEM 2: JEWELRY BOX CONSTRUCTION

The art teacher has assigned a jewelry box construction project. Each student receives two sheets of tin for this project, one sheet for the box and the other for the lid of the box. The sheets of tin you are given are both 24 centimeters by 17 centimeters. Give the dimensions of the box with the largest volume that you can construct from these pieces of tin.

A. Using centimeter graph paper, determine how you will form a rectangular prism from the sheet metal.
B. Write algebraic expressions to represent the length, width, and height of the box.
C. Write a formula to find the volume of the box.
D. Determine the dimensions that will maximize the volume of the box.

MATERIALS

Casio CFX-9850Ga Plus or ALGEBRA FX2.0 Graphing Calculator

Centimeter graph paper

Scissors

Tape

ONE SOLUTION TO PROBLEM 2: JEWELRY BOX CONSTRUCTION

A. Using centimeter graph paper, determine how you will form a rectangular prism from the sheet metal.

Cut squares of equal area from the four corners of the graph paper. Fold the graph paper to make a box without a top.

B. Write algebraic expressions to represent the length, width and height of the box.

Let x = the height of the box in centimeters.

$24 - 2x$ = the length of the box.

$17 - 2x$ = the width of the box.

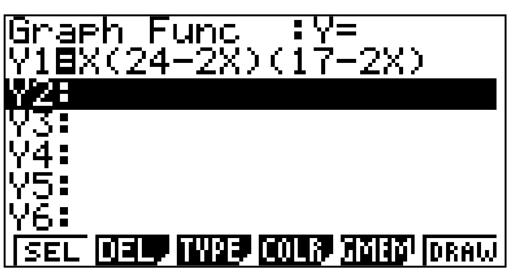
C. Write a formula to find the volume of the box

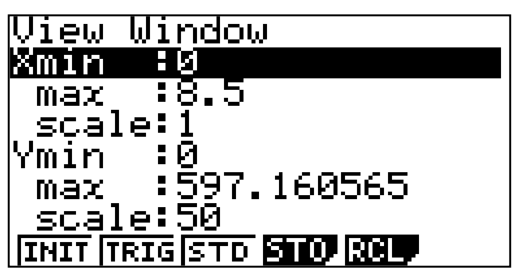
$V = x(24 - 2x)(17 - 2x)$

D. Determine the dimensions that will maximize the volume of the box.

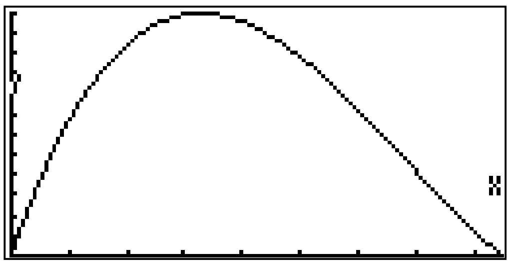
From the MAIN MENU, choose "Graph." Then,

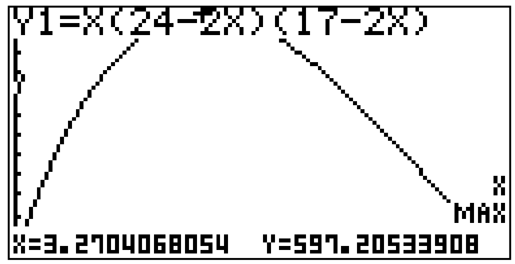
- Type the formula into Y1 and press [EXE] . See below left.
- Press [SHIFT] [F3] to set the window. Only positive values for x should be considered. If $x > 8.5$, the volume would be negative, so the domain should be restricted to $0 \le x < 8.5$. Type in the appropriate values for x, pressing [EXE] after each entry and [EXIT] when finished. This time we'll let the calculator set values for y.
- Press [F6] to draw the graph. Press [F2] followed by [F5] to set the window automatically for y. See below right to see the window that was set for us.





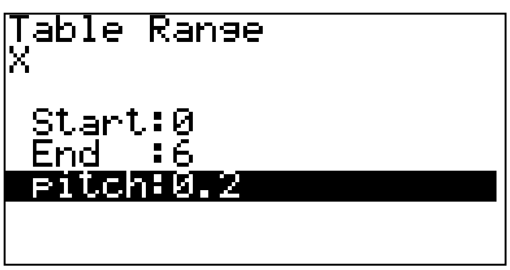
- To return to the graph, press EXIT F6 . See below left.
- To find the x-value that produces the maximum volume, press F5 for the graph solver and F2 for maximum. The screen below right tells us that to maximize the volume, we should cut squares from the corners with sides of 3.27 centimeters. This will form a box with a volume of 597.2 cubic centimeters.

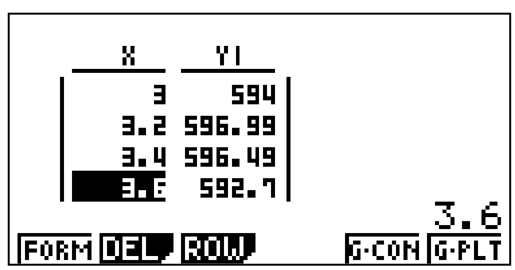




If we were more interested in rounded values, we could have inspected a table to see how changing the sides of the squares we cut out affect the volume. From the MAIN MENU, choose “Table.”

- Press F5 to set the range. Perhaps start at 0 and end at 6, using a pitch of perhaps .2. See below left.
- Press EXIT to leave the range screen and F6 to see the table. Use the down arrow key to find the greatest volume. See below right. This tells us that we most likely will obtain our maximum volume when we cut squares that have a length of somewhere between 3.2 and 3.4 centimeters.





PROBLEM 3: ALCOHOL CONCENTRATION

The polynomial function $a(x) = -0.0915x^3 + 1.771x$ gives the approximate alcohol concentration in hundredths of a percent in an average person's bloodstream x hours after one drink.

A. After how many hours is the alcohol concentration the greatest? What is the maximum alcohol concentration?

B. How long does it take for the alcohol level to return to 0?

C. In some states, a person is legally intoxicated if the blood alcohol level exceeds 0.08%. How long would it take an average person to be considered legally drunk after consuming one alcoholic drink?

PROBLEM 4: AUTOMOBILE EMISSIONS

The number of parts per million of nitric oxide emissions from a particular car engine is approximated by the model $y = -5.05x^3 + 3857x - 38{,}411.25$, for $13 \leq x \leq 18$, where x is the air-fuel ratio.

A. What is the maximum nitric oxide emission for this car engine?

B. What air fuel ratio produces an emission of 2000 parts per million?

C. Why do you suppose there are two ratios that produce the same amount of emissions?

REFERENCES:

PROBLEM 1: CONES FOR SHAPED ICE. *Algebra in a Technological World,* NCTM Addenda Series, 1995.

PROBLEM 3: ALCOHOL CONCENTRATION. *Secondary Math, An Integrated Approach, Focus on Advanced Algebra* Addison-Wesley Publishing Company, 1996.

PROBLEM 4: AUTOMOBILE EMISSIONS. *Precalculus,* Third Edition, Larson, Hostetler, D.C. Heath and Company, 1993.

TEXT SECTION CORRESPONDENCES

The materials in this module are compatible with the following sections in the listed texts.

TEXT	SECTION
AWSM – Focus on Algebra (1998)	9.1
AWSM – Focus on Advanced Algebra (1998)	8.1
Glencoe – Algebra 1 (1998)	9.4
Glencoe – Algebra 2 (1998)	8.1, 8.3, 8.5
Holt Rinehart Winston – Algebra (1997)	
Holt Rinehart Winston – Advanced Algebra (1997)	6.1, 6.4, 6.5
Key Curriculum – Advanced Algebra Through Data Exploration	10.5, 10.6
Merrill – Algebra 1 (1995)	
Merrill – Algebra 2 (1995)	
McDougal Littell – Algebra 1: Explorations and Applications (1998)	10.2
McDougal Littell – Heath Algebra 1: An Integrated Approach (1998)	
McDougal Littell – Algebra: Structure and Method Book 1 (2000)	4.9
Prentice Hall – Algebra (1998)	10.2, 10.7
Prentice Hall – Advanced Algebra (1998)	6.2, 6.3, 6.4
SFAW: UCSMP – Algebra Part 1 (1998)	
SFAW: UCSMP – Algebra Part 2 (1998)	10.5, 13.6
SFAW: UCSMP – Advanced Algebra Part 1 (1998)	
SFAW: UCSMP – Advanced Algebra Part 2 (1998)	11.2
Southwestern – Algebra 1: An Integrated Approach (1997)	11.8, 12.7, 10.6