Balancing Chemical Equations

Introduction: Chemical reaction equations are mathematical and symbolic models of real world compound and events. As such, they are governed by certain rules. One rule is called the Law of the Conservation of Matter, that whatever amount of matter you start with before a reaction must be the same amount you finish with after the reaction. Because of this law, chemical reactions can be thought of as linear equations and mathematically manipulated as such. This lesson helps students to use matrices to balance chemical equations.

Objectives: Students will be able to…
1. Follow procedures to balance chemical equations.
2. Input data in a graphing calculator.
3. Discuss applications of results.

Related Key Words: balanced equation single replacement reaction product
double replacement reaction synthesis reaction compound
conservation of matter reactants element

Materials: Balancing chemical reactions worksheet
Paper
9850 / 9850+ graphing calculator

Purpose: To balance chemical reactions using matrices.

STEP 1— Consider the follow simple equation:
\[ \text{H}_2 + \text{O}_2 \rightarrow \text{H}_2\text{O} \]
We need to assign a variable to each of the terms of the reaction equation.
\[ a\text{H}_2 + b\text{O}_2 \rightarrow c\text{H}_2\text{O} \]

STEP 2-- For each element we must create an equation based on the number of the atoms of that element in the compound, such as for hydrogen:
\[ 2a \text{H} + 0b \text{O} = 2c \text{H} \]
(On the right, there are 2 atoms of hydrogen in the hydrogen gas, on the left there are 2 atoms of hydrogen in the water compound. So actually, the hydrogen atom is balanced as the equation is written.)

STEP 3-- Now we create an equation for the oxygen atom:
\[ 0a \text{H} + 2b \text{O} = 1c \text{O} \]
(Here we start with 2 atoms of oxygen atoms, but finish with only one. The conservation of matter says this is not possible, therefor the equation is not balanced.)

STEP 4-- These equations are rewritten without the elements:
\[ 2a + 0b = 2c \]
\[ 0a + 2b = 1c \]
We will solve the two equations for an integer value for c.

STEP 5-- Matrix \( A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \) Matrix \( B = \begin{bmatrix} a \\ b \end{bmatrix} \) Matrix \( c = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)
STEP 6-- Then the matrix equation is written as:
Matrix A x Matrix B = c (Matrix C)
Therefor the equation for Matrix B is:
Matrix B = (Matrix A)^{-1} x c (Matrix C)

STEP 7— Now we need to input the data into the calculator. From the main menu, select the “MAT” icon by highlighting it and pressing the “EXE” key, or press “3” on the number pad. (See below)

![Calculator Screen with Matrix Entries]

Count the size of the Matrix A from the equation. It will be a 2x2 matrices. Press the “2” then “EXE” and repeat to create a 2x2 matrix. Repeat the process for Matrix B and Matrix C, making them each a 2x1 matrices. (See above)

STEP 8— With the data now inputted to the matrices, return to the run menu. Now the manipulations of the matrices will take place. Press the “OPTN” (next to the yellow shift key). This will allow us to select the matrices we wish to manipulate. Press the F2 key under “MAT”. First, we will determine the integer value of c from the determinant of Matrix A.
Press F3 for “Det”, followed by F1 and then “Alpha” and the “A” key. Next use the “à” key above the “AC/ON” key followed by pressing the “ALPHA” key and “C”. This will calculate the integer value for “c” which we will use later. Your screen should now show:

Det Mat A → C (See above)

STEP 9— The result will be the values that correspond to the variables in Matrix B:

![Calculator Screen with Matrix B Result]

Substituting the values for a, b, and c, we get the following equation:

$$4H_2 + 2O_2 \rightarrow 4H_2O$$

This result is not entirely correct. We should factor out a 2 to make the equation:

$$2H_2 + O_2 \rightarrow 2H_2O$$

The extra factor of 2 is a result of the fact that the hydrogen atoms were already balanced. Chemical equations should be reduced to lowest common factors.
Sample Problem:
Balance the following chemical equation:

\[ \text{Cu}_2\text{S} + \text{HNO}_3 \rightarrow \text{Cu(NO}_3)_2 + \text{CuSO}_4 + \text{NO}_2 + \text{H}_2\text{O} \]

Step 1: Assign variables to each compound:

\[ a\text{Cu}_2\text{S} + b\text{HNO}_3 \rightarrow c\text{Cu(NO}_3)_2 + d\text{CuSO}_4 + e\text{NO}_2 + f\text{H}_2\text{O} \]

Step 2: Create an equation for each of the elements:

\[ 2a\ Cu = c\ Cu + d\ Cu \]
\[ aS = dS \]
\[ bH = 2fH \]
\[ bN = 2eN + eN \]
\[ 3bO = 6eO + 4dO + 2eO + fO \]

Step 3: Rewrite the equations without the elements and solve them for f:

\[ 2a + 0b - 1c - 1d + 0e = 0f \]
\[ 1a + 0b + 0c - 1d + 0e = 0f \]
\[ 0a + 1b + 0c + 0d + 0e = 2f \]
\[ 0a + 1b - 2c + 0d - 1e = 0f \]
\[ 0a + 3b - 6c - 4d - 2e = 1f \]

Step 4: Set up our matrices:

Step 5: Determine the integer value for f and solve for Matrix B.

\[ \text{Cu}_2\text{S} + 12\text{HNO}_3 \rightarrow \text{Cu(NO}_3)_2 + \text{CuSO}_4 + 10\text{NO}_2 + 6\text{H}_2\text{O} \]

This activity was developed by E. William Turley, Kiski School, Mathematics Department.
Questions and Problems:

   Level 1: Answer the following questions in complete, well-structured sentences.

1. Define double replacement reaction.
2. What are the reactants of a chemical reaction?
3. What are the products of a chemical reaction?
4. Explain how the conservation of matter relates to creating balanced chemical equations.
5. How would you categorize the first sample equation (2H₂ + O₂ \rightarrow 2H₂O)?

   Level 2:

1. Explain the difference between “\rightarrow” and “\leftrightarrow” in a chemical reaction.
2. How would you categorize the second sample equation?
3. What does a catalyst do compared to an inhibitor? Give two examples of each.
4. Discuss the differences between an atom, an element, and a compound. Give examples for each.
5. Balance and categorize the following equation:
   \[ \text{CaCl}_2 + \text{H}_2\text{O} \rightarrow \text{Ca(OH)}_2 + \text{HCl} \]

Extension:

Have students select their own equations to balance for each other using the matrix technique to solve them.