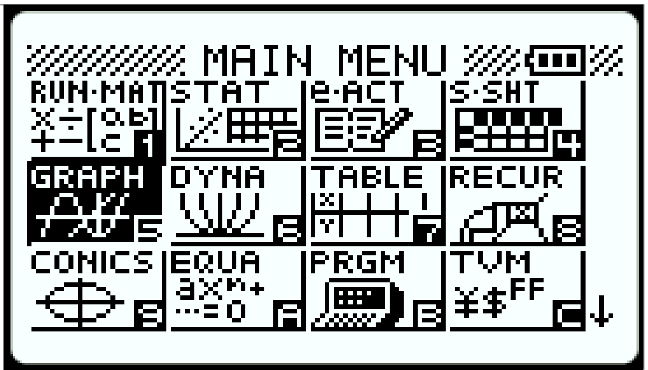
Unit 7: Quadratic Equations	
Scientific Calculator Required	Lessons 8,13,16,17,18
Graphing Technology Recommended	Lessons 3,4,18,21,24
Graphing Technology Required	Lessons 5,9,10,15,20,22

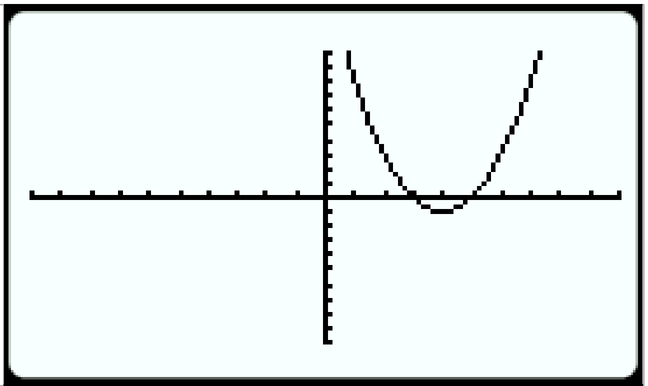
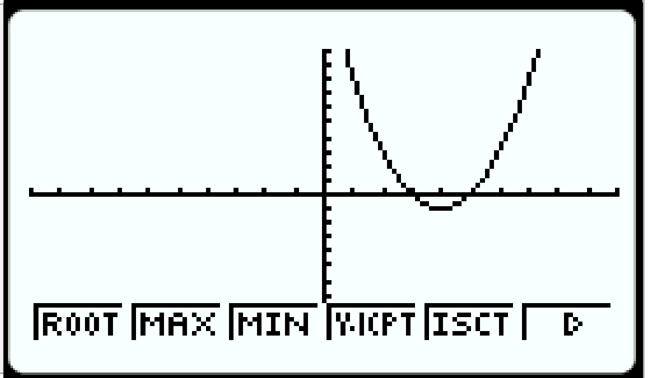
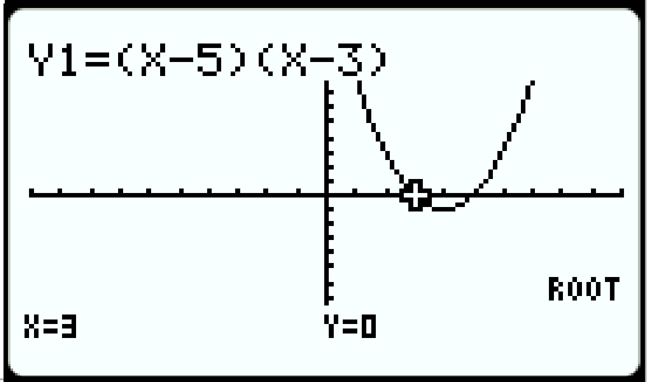
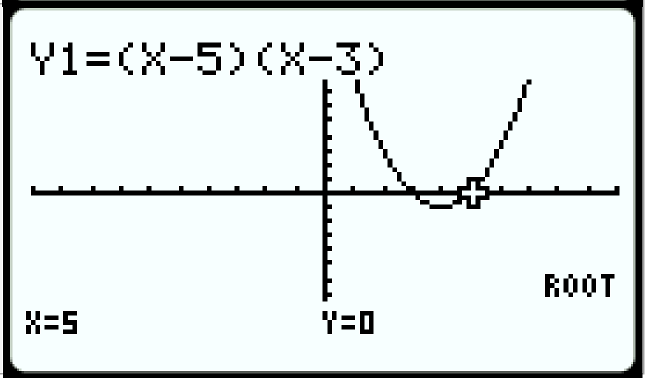
Lesson 5 – How to Graph Quadratic Functions (Factored Form)

(Example: IM Lesson 5.2: Solving by Graphing.)

1. To graph a quadratic equation go to the **Graph App**; press [MENU] , [5] - [GRAPH].

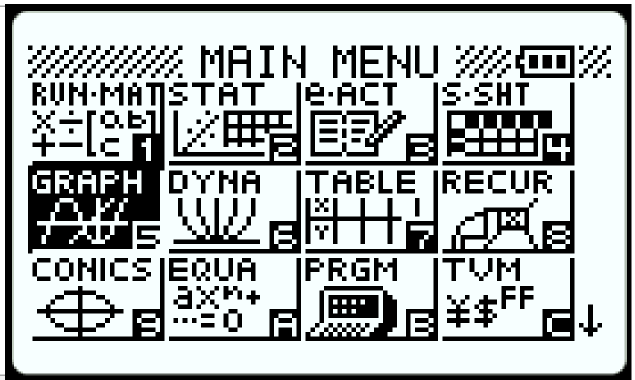
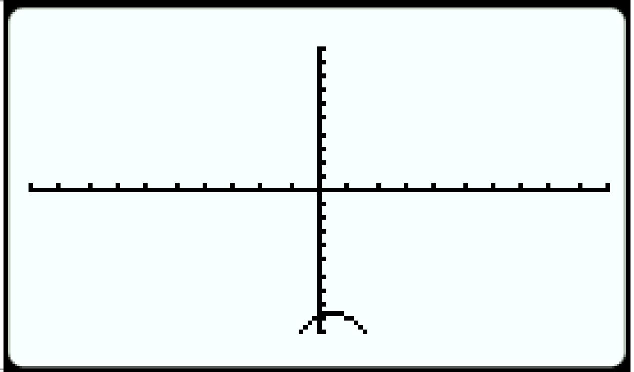
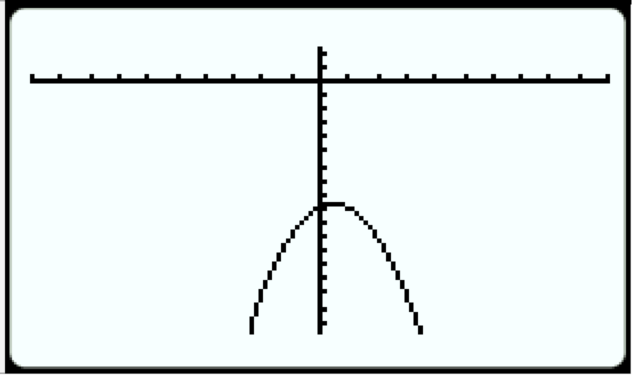


2. Enter the function **(*x* – 5)(*x* – 3)** into the first line for **Y1**. Press [EXE] when you finish.

3. Press F6 – DRAW to view the graph of the function.	
4. To find the ***x*-intercepts**, press F5 **– G-Solv** to see the **Graph-Solve** options.	
5. Press F1 – ROOT. This will display the first ***x*-intercept** at the bottom of the screen.	
6. Press the right arrow ▶ to go to the next **root (*x*-intercept).** The ***x*-intercepts** can also be called **roots, zeros**, or the **solution** to ***f*(*x*) = 0**.	

Lesson 6 – Rewriting Quadratic Expressions in Factored Form (Part 1)

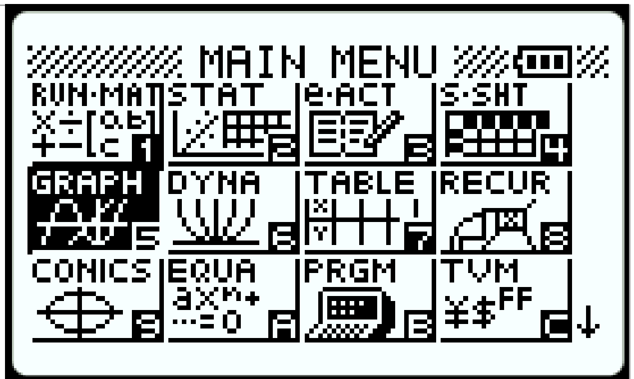
(Example: IM Lesson 6: Practice Problem 8)

1. To graph a quadratic equation, go to the **Graph App**; press [MENU], [5] - [GRAPH].	
2. Enter the function **(2 – *x*)(*x* + 3) – 11** into the first line for **Y1**. Press [EXE] when you finish.	
3. Press [F6] – [DRAW] to view the graph of the function. If your window is in the **standard window** you may just see the **turning point** of the **parabola**, as shown to the right.	
4. To quickly fix this, press the down arrow. This will shift the graph window up to show more of the function. Since the graph does not cross the ***x*-axis,** there are no real solutions. **<u>Note:</u>** All 4 **arrow keys** will shift the viewing window respectively.	

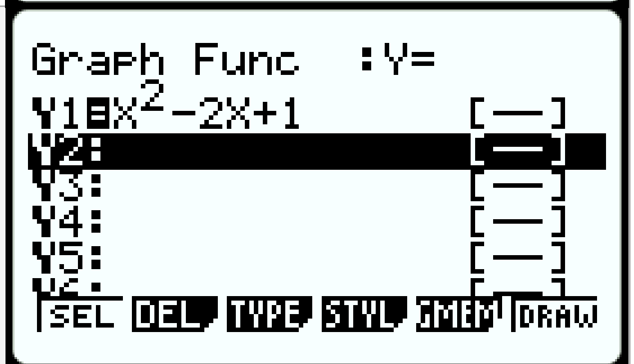
Lesson 9 – Solving Quadratic Equations by Using Factored Form

(Example: IM Lesson 9.3: Revisiting Quadratic Equations with Only One Solution)

1. To graph a quadratic equation go to the **Graph App**; press MENU, 5 - GRAPH.

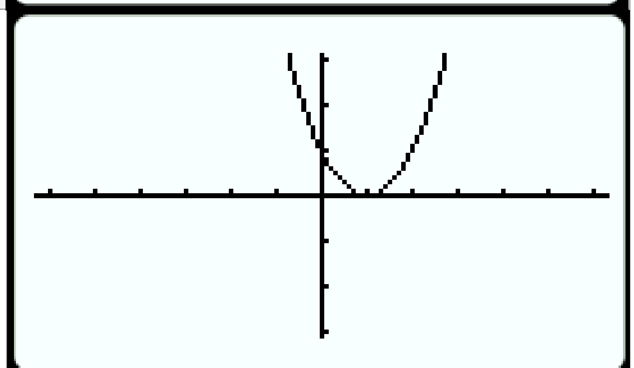


2. Enter the function ***x*² – 2*x* + 1** into the first line for **Y1**. Press EXE when you finish.

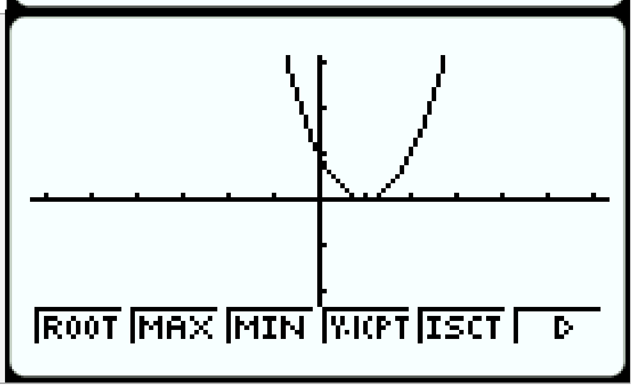


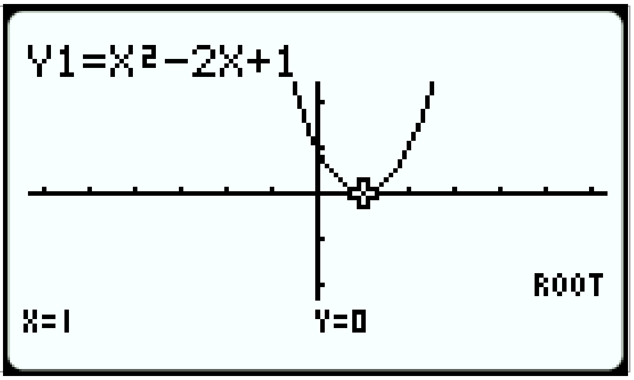
3. Press F6 – DRAW to view the graph of the function.

   What do you notice about the ***x*-intercepts** of the graph? What do the ***x*-intercepts** reveal about the function?



4. To find the ***x*-intercepts**, press F5 **– G-Solv** to see the **Graph-Solve** options.

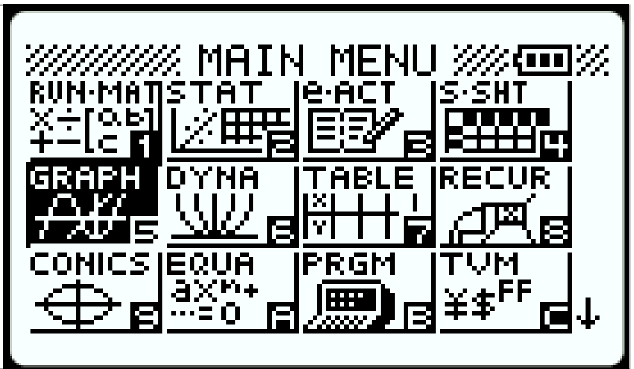


5. Press [F1]–[ROOT]. This will display the first ***x*-intercept** at the bottom of the screen. In this example, there is only one **“double”** root at ***x* = 1**.	

Lesson 10 – Rewriting Quadratic Expressions in Factored Form (Part 4)

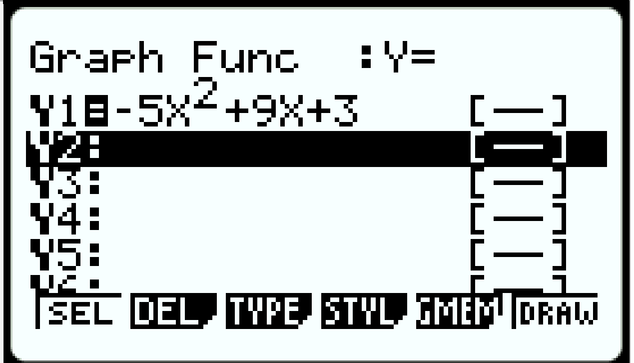
(Example: IM Lesson 10.3: Timing a Blob of Water)

1. Students are asked to find the time the water droplet is in the air. Soon they will find that factoring the equation set equal to zero will not always work. However, the ***x*-intercept** can be found using the **root solve tool** on a graph. To graph the quadratic equation**,** go to the **Graph App**; press [MENU], [5] - [GRAPH].



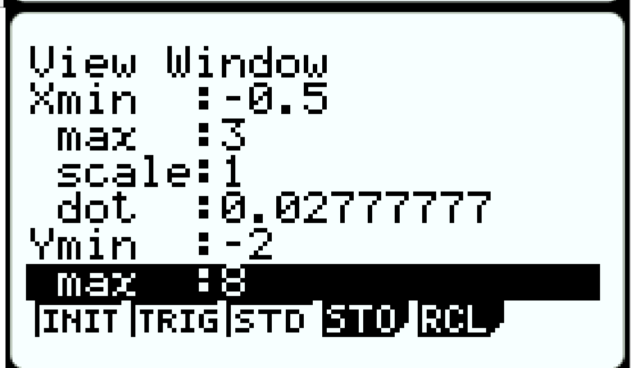
2. Enter the function **-5*x*^{2} + 9*x* + 3** into the first line for **Y1**. Press [EXE] when you finish.

   The equation in the example uses "***x***" instead of "***t***" but when using the graphing feature in the calculator, use "***x***" for the input variable.

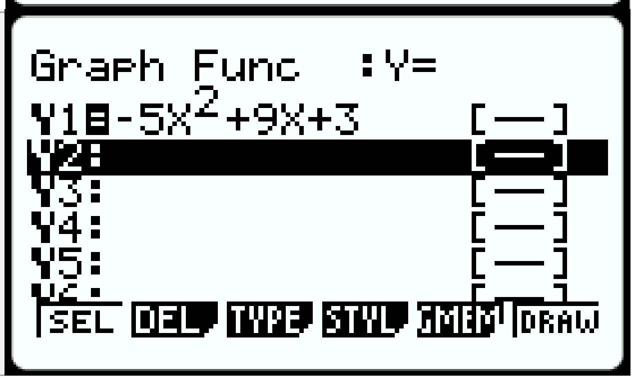


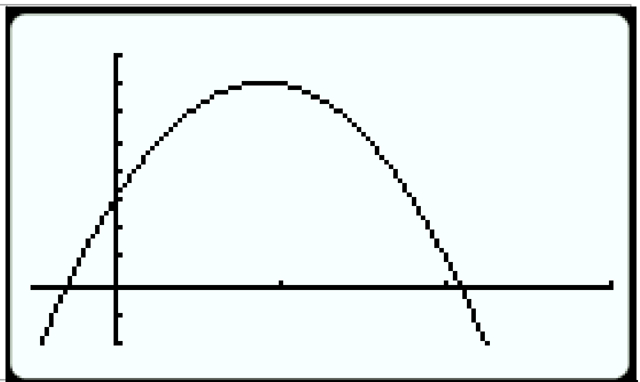
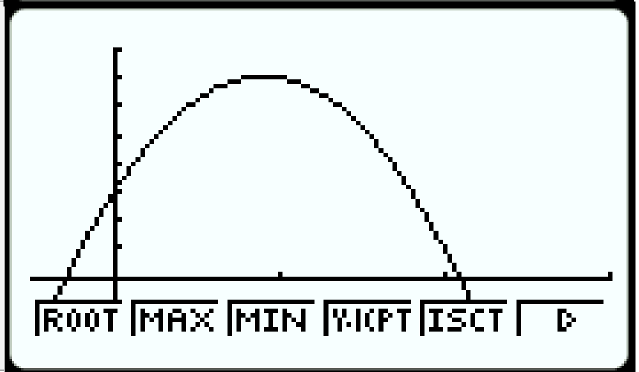
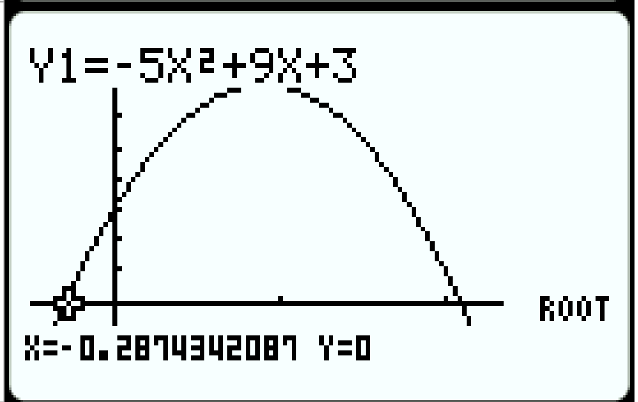
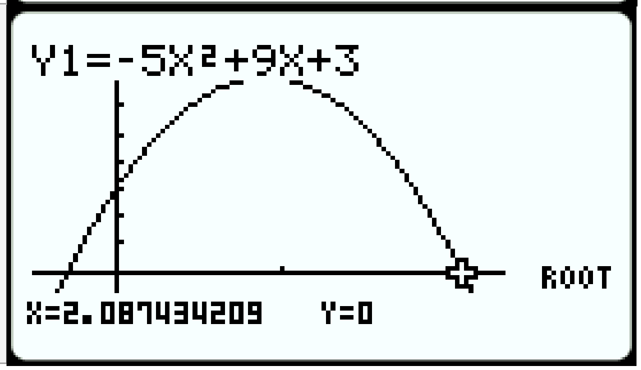
3. Since this application deals with positive time and positive heights, adjust the viewing window by pressing [SHIFT] [F3] – [V-WIN].

   Adjust the window to the values shown to the right.



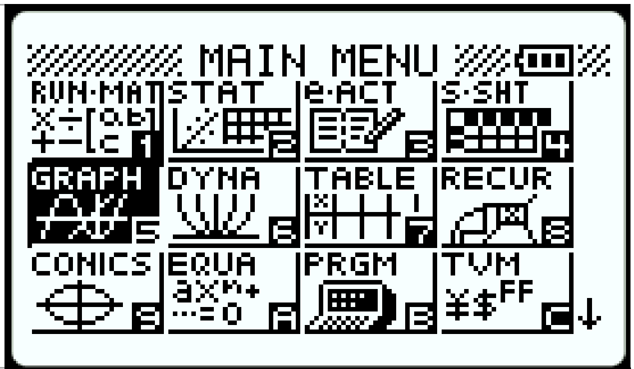
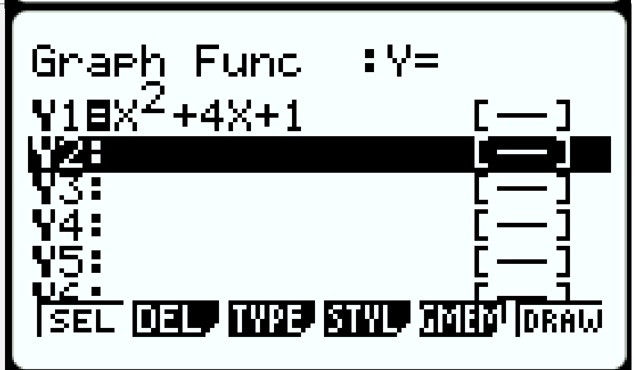
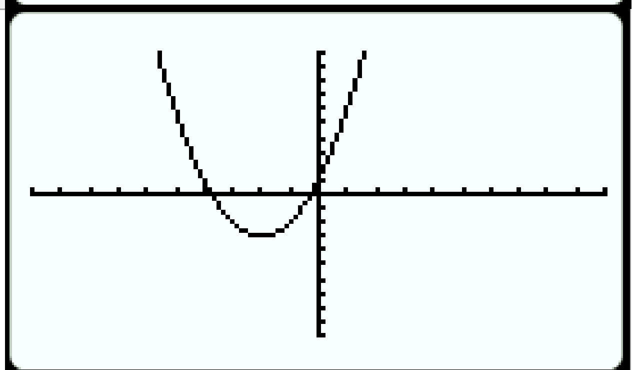
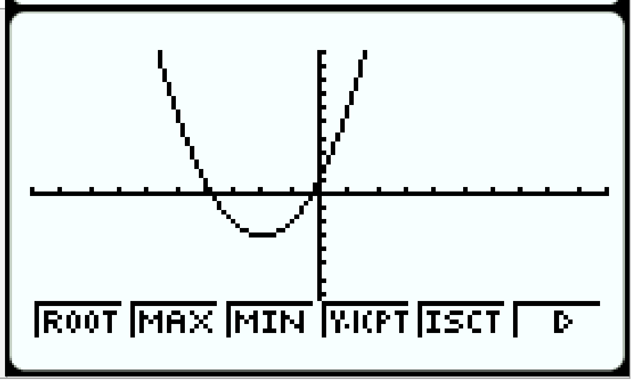
4. When finished, press [EXE] again to return to the graph function entry window.



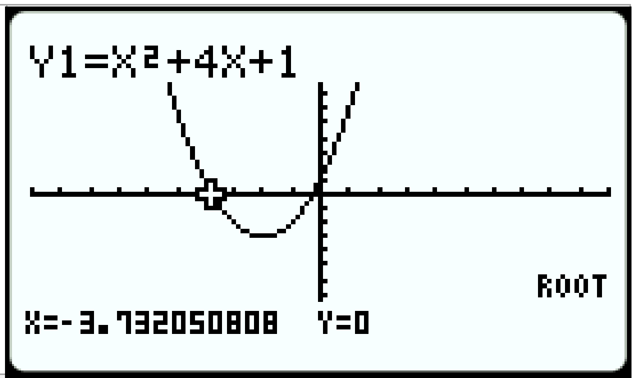
5. Press F6 – DRAW to view the graph of the function.	
6. To find the ***x*-intercepts**, press F5 – G-SLV to see the **Graph-Solve** options. The ***x*-intercepts** can also be called **roots, zeros**, or the **solution** to ***f*(*x*) = 0**.	
7. Press F1 – ROOT. This will display the first ***x*-intercept** at the bottom of the screen. Since this time is negative, this value is **not valid** as time before the water nozzle shoots at 0 seconds is outside the **domain** of our model; ***t* ≥ 0**.	
8. Press the right arrow ▶ to go to the next **root.** This value will represent the time the water droplet hits the ground, as ***h*(*t*)** will be **0 meters** at this time **of 2.087 seconds**.	

Lesson 15a – Verify Approximate Solutions to Quadratics Graphically

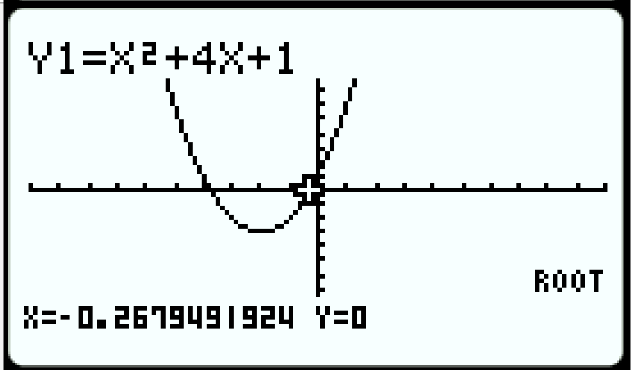
(Example: IM Lesson 15.3: Finding Irrational Solutions by Completing the Square)

1. To **verify solutions** to a quadratic equation solved by **completing the square**, we can graph the function and find the **roots of the equation** to find an **approximate** solution. To graph the quadratic equation**,** go to the **Graph App**; press MENU , 5 - GRAPH.	
2. To verify the first question, enter the function $x^2 + 4x + 1$ into the first line for **Y1**. Press EXE when finished.	
3. Press F6 – DRAW to view the graph of the function.	
4. We can see the graph crosses the ***x*-axis** twice; indicating 2 **solutions**. To find the ***x*-intercepts**, press F5 **– G-Solv** to see the **Graph-Solve** options. Recall, the ***x*-intercepts** can also be called **roots, zeros**, or the **solution** to $f(x) = 0$.	

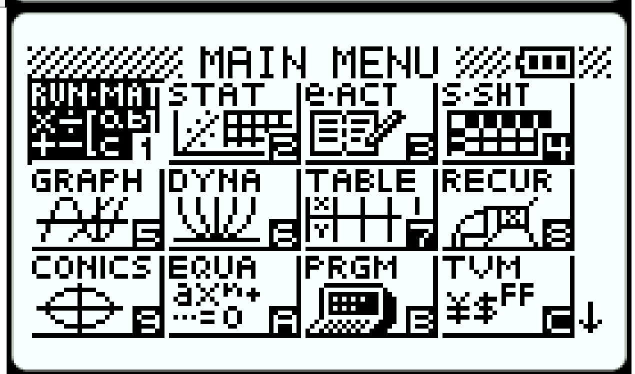
5. Press [F1] – [ROOT]. This will display the first ***x*-intercept** at the bottom of the screen. The first **root** is approximately **-3.732**.



6. If you press the right arrow (▶), you will see the second **root** at the bottom of the screen which is approximately **-0.268**.

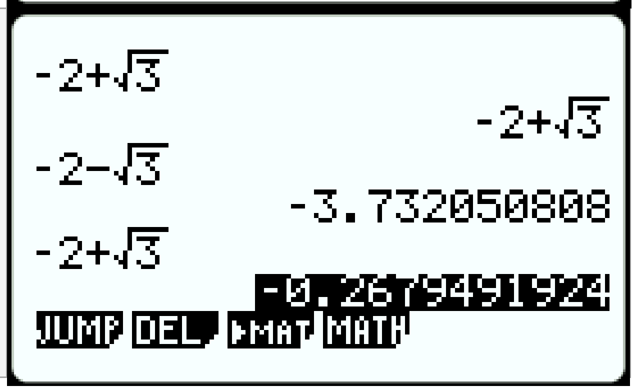


7. To view the **exact solutions** from completing the square as equivalent **decimal approximations** press [MENU], [1] - [RUN·MAT]. to go to the **Run-Matrix App**.



8. Enter the **exact solution** found by **completing the square** into the calculator as shown. Each solution needs to be entered into the calculator separately. If your calculator still shows the **exact value**, highlight the value and press the [S⇔D] button to quickly switch to the **decimal approximation**.

   Graphically, **-3.732** verifies $-\mathbf{2}-\sqrt{\mathbf{3}}$ while the other root, **-0.268** verifies $-\mathbf{2}+\sqrt{\mathbf{3}}$.



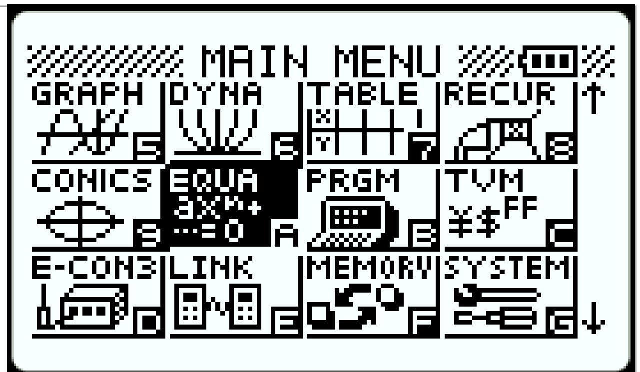
Lesson 15b – Verify Exact Solutions to Quadratics with Equation Solver

(Example: IM Lesson 15.3: Finding Irrational Solutions by Completing the Square)

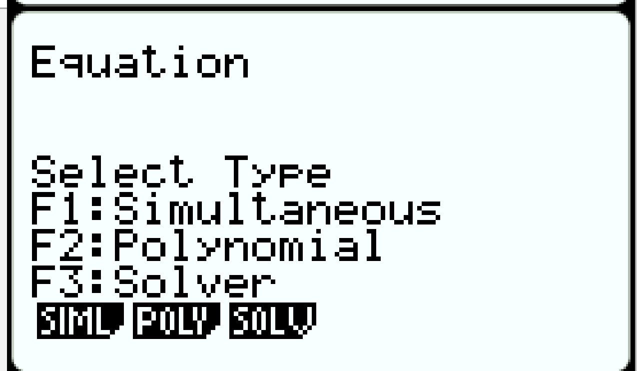
1. To **verify exact solutions** to a quadratic equation solved by **completing the square**, we can use the **polynomial equation solver**.

To use the **polynomial equation solver** to find the exact solutions to an equation**,** go to the **Equation App**; press [MENU], [X,θ,T] – [EQUA].

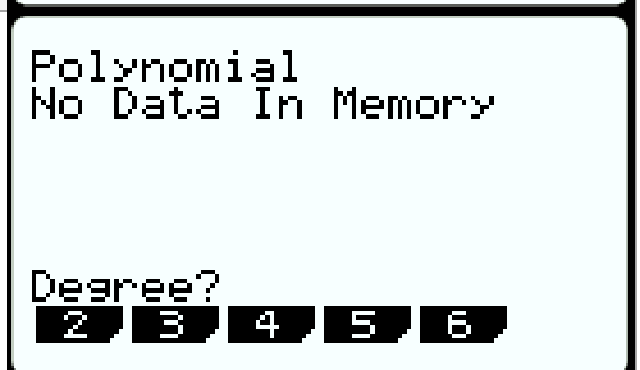
Note: After pressing [MENU], you do not need to press [ALPHA] to select a letter choice.



2. Since a **quadratic equation** is a type of **polynomial equation**, press [F2] – [POLY].

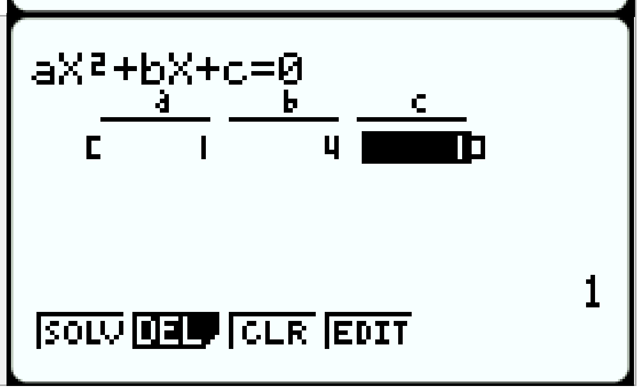


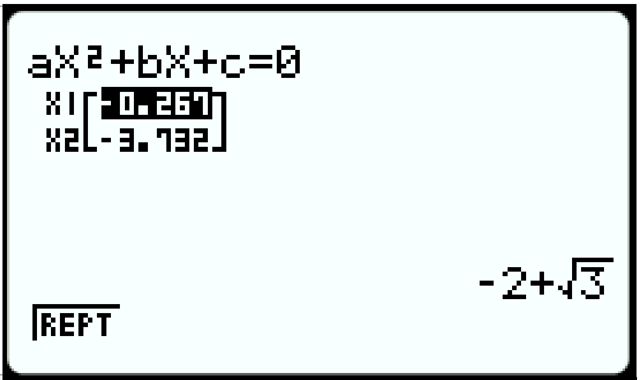
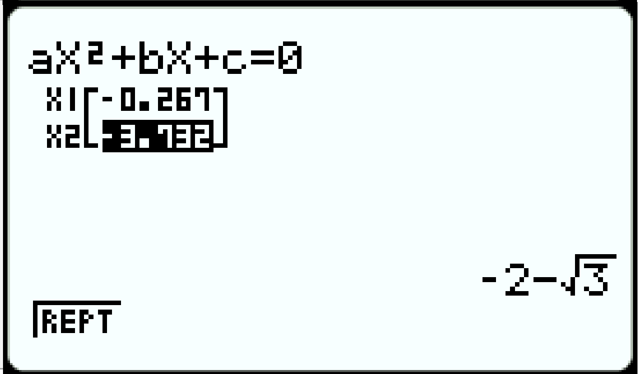
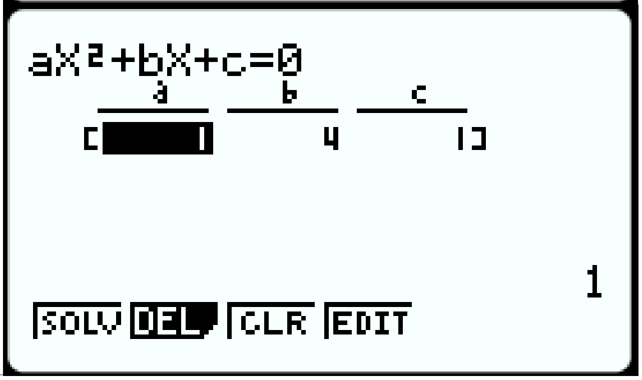
3. A **quadratic** equation is a **second-degree polynomial** equation as the **highest exponent** is a **2**. Press [F1] – [2].



4. To verify the first equation, $x^2 + 4x + 1 = 0$, enter its **coefficients**; **1** for ***a***, **4** for ***b***, and **1** for ***c***; into the **polynomial solver**. Press **1**, [EXE], **4**, [EXE], and **1** for this example.

Remember to first set any polynomial **equal to zero** to enter the correct value for the **constant term; *c*** for a second-degree polynomial.



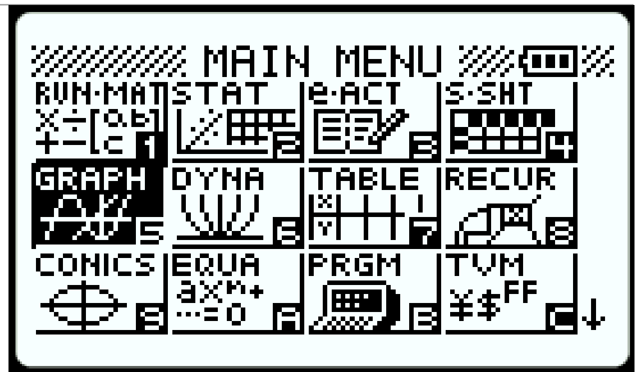
5. After typing the **1** for ***c***, press either EXE or F1 – SOLV to run the solver as shown to the right. **Both solutions** are shown as **decimal approximations** in the top left of the screen. The value highlighted will display **the exact value** in **simplest radical form** in the bottom right of the screen.	
6. Press the down arrow (▼) or EXE to display the **exact value** for the **second root**.	
7. To solve another quadratic equation, press EXE, or EXIT, or F1 – REPT to **repeat** the process of entering new **coefficients**.	

Lesson 15c – Graphing to Approximate Irrational Solutions

(Example: IM Lesson 15: Practice Problem 4)

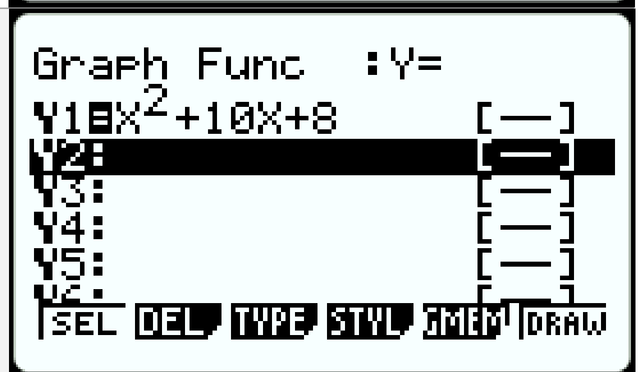
1. To find **approximate solutions** to a quadratic equation, we can graph the function and find its **roots**.

   To graph the quadratic equation**,** go to the **Graph App**; press MENU , 5 - GRAPH.

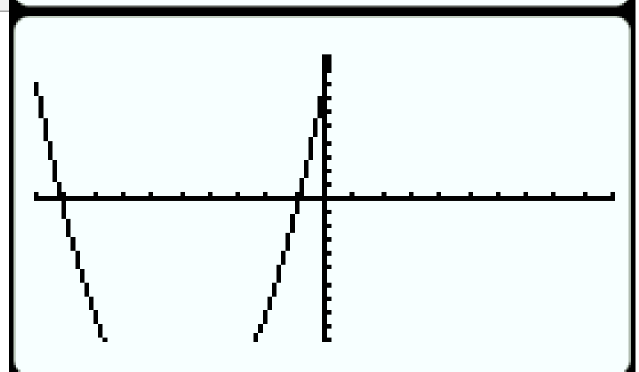


2. To find the **approximate solutions** for the first question, enter the function x^2 **+ 10*x* + 8** for **Y1**.

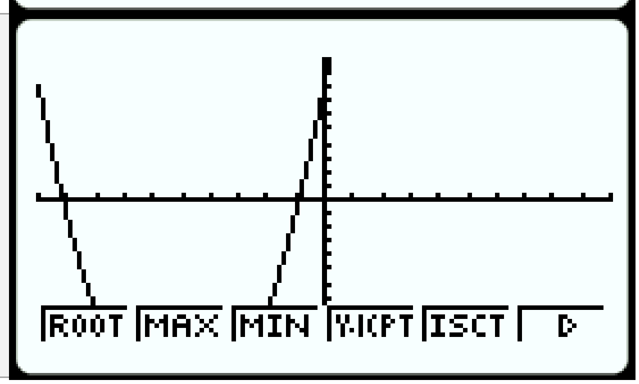
   Press EXE when finished.

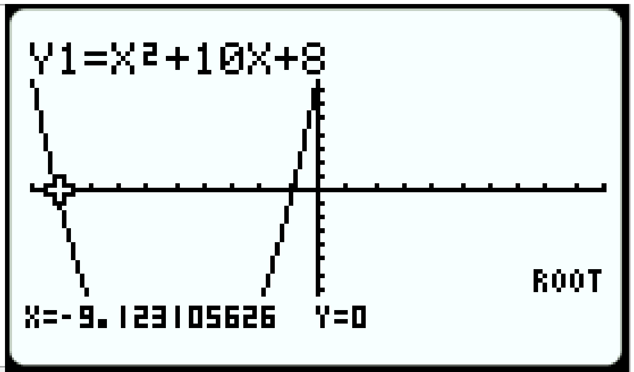
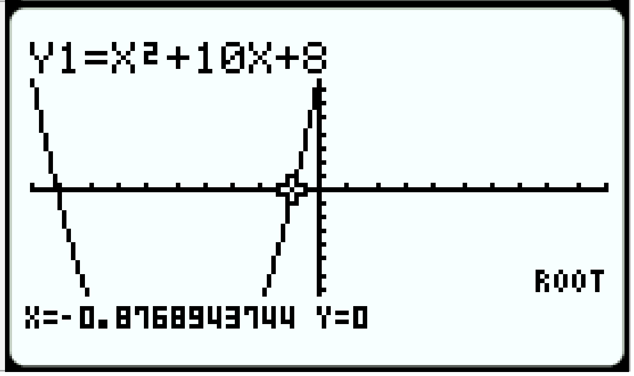


3. Press F6 – DRAW to view the graph of the function.



4. From the graph, we can see that it crosses the ***x*-axis** twice. To find the **roots/solutions**, press F5 **– G-Solv** to view the **Graph-Solve** options.

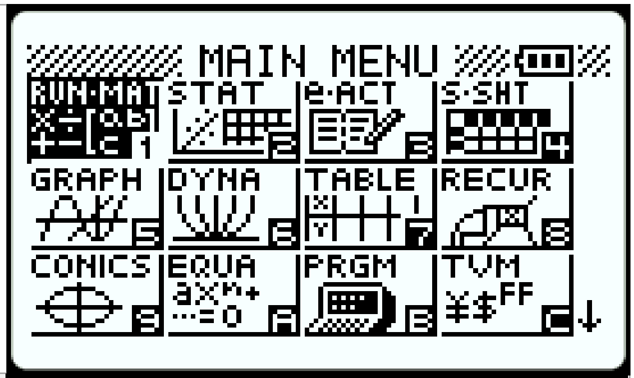


5. Press F1 – ROOT. This will display the first **root** at the bottom of the screen. The **first solution** for this equation is **approximately -9.123**.	
6. If you press the right arrow (▶), you will see the second **root** at the bottom of the screen which is approximately **-0.877**. This value is a **second solution** to the first equation; $\boldsymbol{x^2 + 10x + 8 = 0}$. You can find the **exact solutions** by **completing the square** algebraically.	

Lesson 16a – Simplify the Quadratic Formula by Manual Substitution

(Example: IM Lesson 16.3: Meeting the Quadratic Formula)

1. Press [MENU], [1] - [RUN·MAT]. to go to the **Run-Matrix App**.



2. We can use this section of the calculator to simplify the quadratic formula. To solve the equation: $x^2 + 4x - 5 = 0$, using the quadratic formula we must first identify the a,b and c values. For this equation:

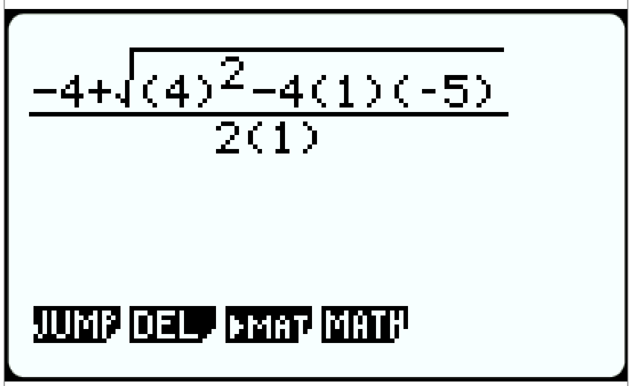
   ***a* = 1** ***b* = 4** ***c* = -5**

   Now we will manually substitute these values into the quadratic formula. In your calculator press the fraction button, [fraction], to start.

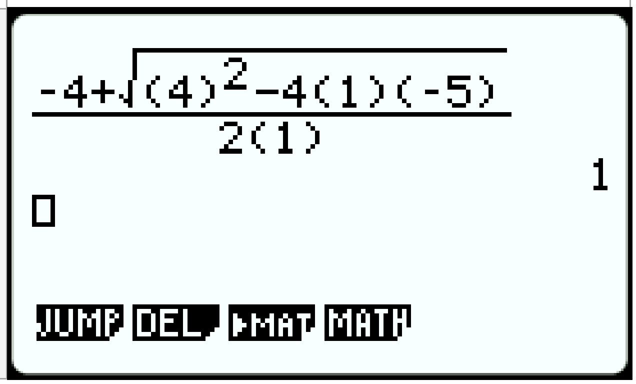
3. The quadratic formula is unique because it contains the **"±" symbol**, yielding 2 answers. This requires using the **addition** and **subtraction** signs **individually** on the calculator.

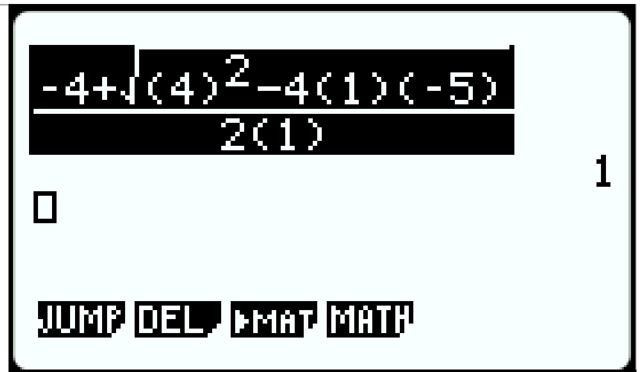
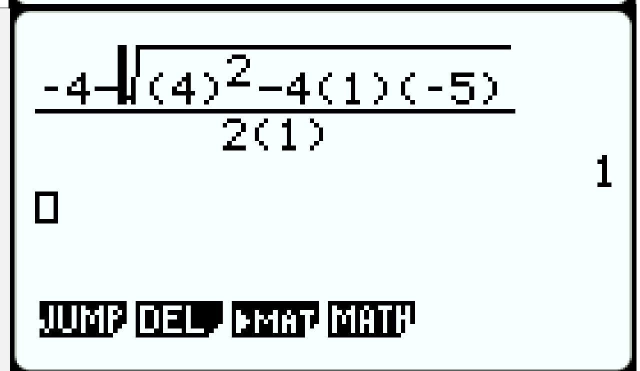
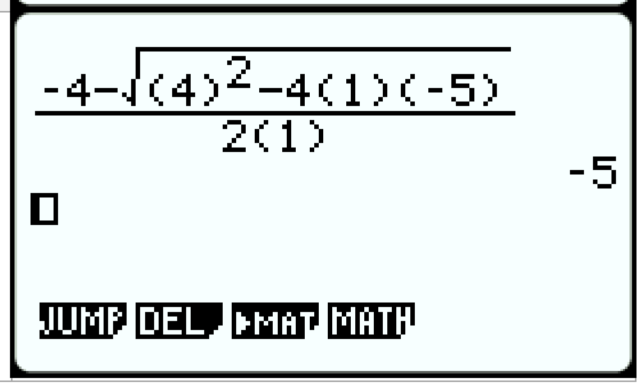
   We can use the plus symbol first then edit the equation in our calculator to use the subtraction sign after.

   To get the **square root symbol** press [SHIFT] then [x^2]. Place ( ) around each value as you manually substitute them into the formula.



4. Once finished, press [EXE] to find **the first solution**, **1**.

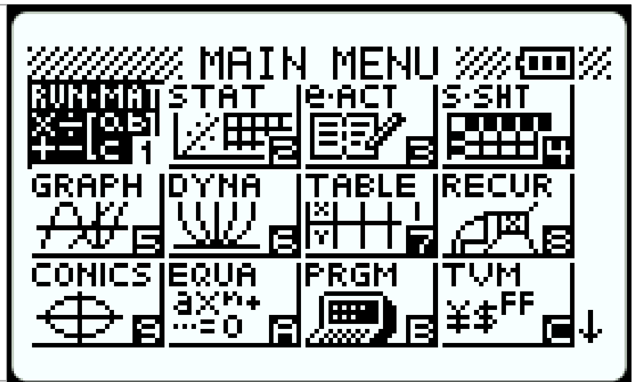


5. Now press the **up arrow** until you highlight the entire **expression** you just typed.	
6. Then press the **right arrow** and **delete** the **addition** sign in the numerator and **replace** it with the **subtraction** sign.	
7. Press EXE to recalculate the expression. You will now see the **1** be replaced with the **second solution** of **-5**.	

Lesson 16b – Simplify the Quadratic Formula by Storing Values

(Example: IM Lesson 16.3: Meeting the Quadratic Formula)

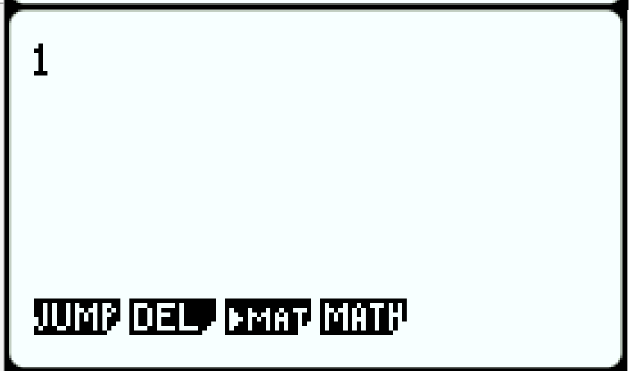
1. Sometimes we want to use the quadratic formula to solve an equation with answers in **simplest radical form**. If we need to repeat this process for multiple questions, we can store the values for ***a***, ***b***, and ***c***, and the calculator will "**auto-refresh**" calculations. Press [MENU], [1] - [RUN-MAT]. to go to the **Run-Matrix App**.



2. We can use this section of the calculator to simplify the quadratic formula. To solve the third equation: $x^2 + 10x + 18 = 0$, we must first identify the ***a***, ***b***, and ***c*** values. For this equation:

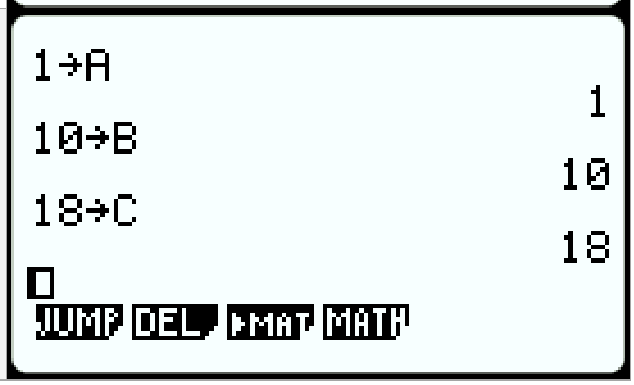
   a **= 1** ***b*** **= 10** ***c*** **= 18**

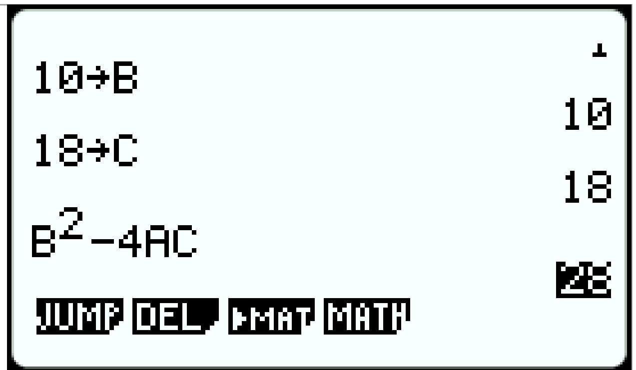
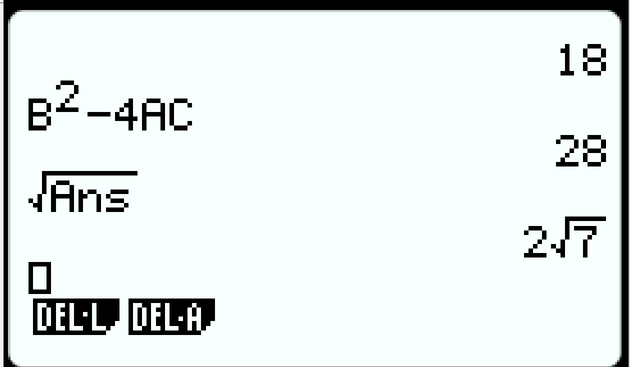
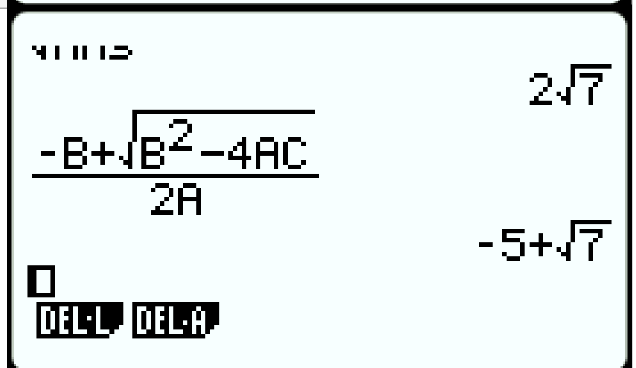
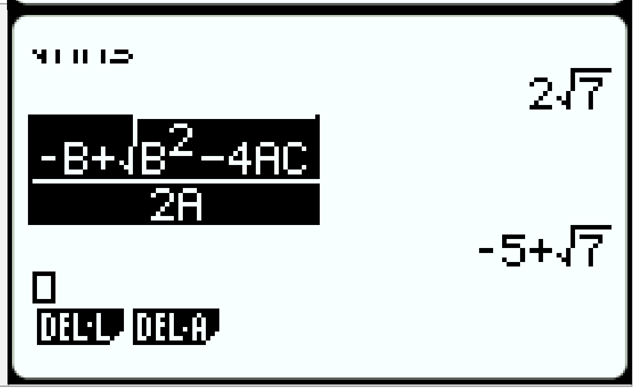
   Now we will **store** these values for ***a***, ***b***, and ***c*** in the calculator. In your calculator, type the value first.



3. Next press [→], which is the Store command. Then use the [ALPHA] button followed by [X,θ,T] to obtain the letter ***a***. Press [EXE].

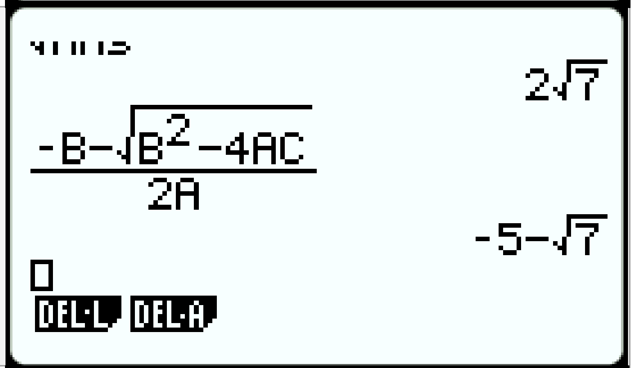
4. Repeat the process to store the values for ***b*** and ***c***.



5. At times, knowing the value of the **discriminant** allows us to know the type of roots and may be necessary to show **simplifying the radical** for the **NYS Regents exam**. Since we stored ***a***, ***b***, and ***c*** in the calculator, we can type $b^2 - 4ac$ to evaluate the discriminant for this quadratic. Since the discriminant is **28**, there will be **2 irrational roots**.	
6. To know if this would **simplify**, press [SHIFT] [x^2] for the **square root** ($\sqrt{\ }$) and then [SHIFT] [(−)] for **answer**. Press [EXE]. **<u>Note:</u>** This intermediate step of $\sqrt{28} = 2\sqrt{7}$ should be shown in student work to earn full points on a 4 pt SA question on the **NYS Regents exam**. (Ref: Jan '25 Q33 Rubric)	
7. Now we will type the entire quadratic formula to find the final, simplified answer. The quadratic formula is unique as it contains the "±" **symbol**, yielding 2 answers. This requires using the **addition** and **subtraction** signs individually on the calculator. We will use the **addition symbol** first while typing the quadratic formula as shown. Press [EXE] when finished.	
8. Now press the **up arrow** (▲) twice until you highlight the entire **expression** you just typed.	

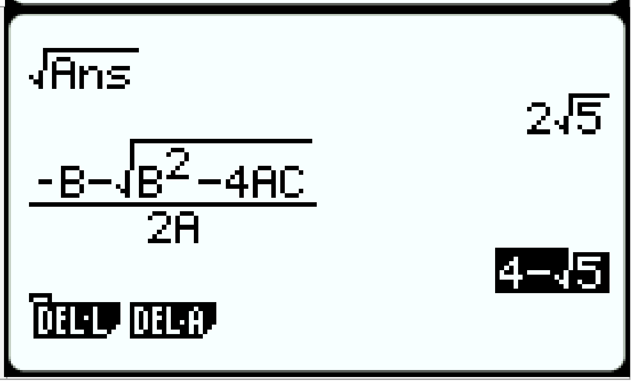
9. Then press the **right arrow** and **delete** the **addition** sign in the numerator and **replace** it with the **subtraction** sign.

10. Press EXE to **recalculate** the expression. You will now see the **addition sign** be replaced with a **subtraction sign** for the second solution.



11. If we had to solve more quadratic equations in standard form, we could **arrow up** and **edit** our **stored values** for ***a***, ***b***, and ***c*** and the calculator will automatically recalculate each of the expressions on the lines below. For the next problem, $x^2 - 8x + 11 = 0$, store: ***a*** **= 1** ***b*** **= 10** and ***c*** **= 18**.
<u>Note:</u> If changing ***b*** or ***c*** results in an **imaginary root**, an error message will result if the calculator is in **real-solution** mode.

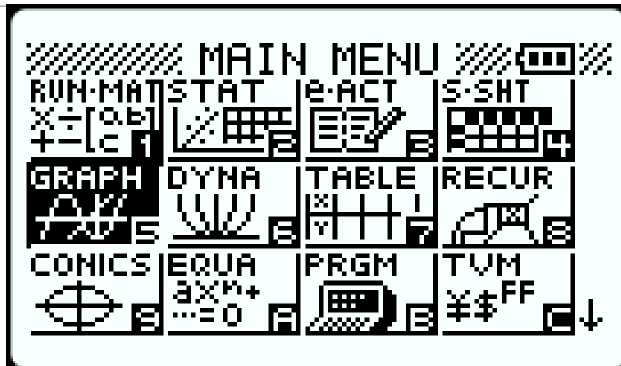
12. **Scrolling down**, we can see that **all** our **expressions** have been **recalculated** with the **updated values** stored for ***a***, ***b***, and ***c***. In simplest radical form, the solutions for the fourth equation are $4 \pm \sqrt{5}$.



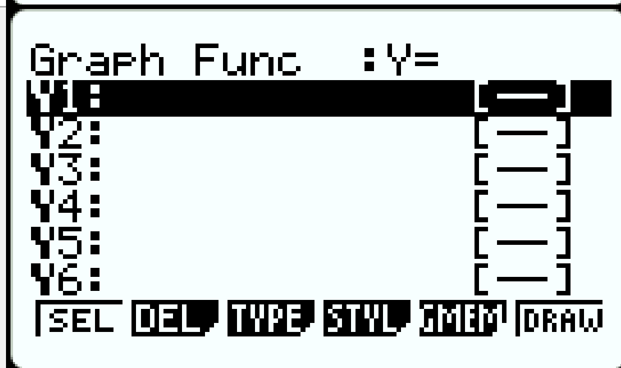
Lesson 16c – Comparing Features of Quadratic Functions Graphically

(Example: IM Lesson 16: Practice Problem 4)

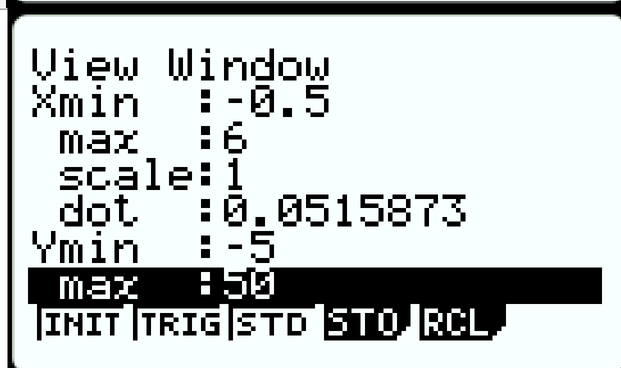
1. Given the equations of two projectiles, we need to determine which will hit the ground first and each object's maximum height. Both can be determined graphically. To graph each equation, press [MENU], [5] - [GRAPH].



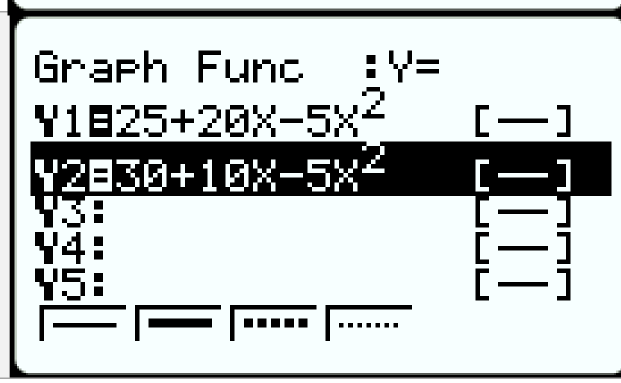
2. Before entering our equations, adjust the viewing window to appropriate dimensions. Press [SHIFT], then [F3] – [V-WIN] to set the viewing window.

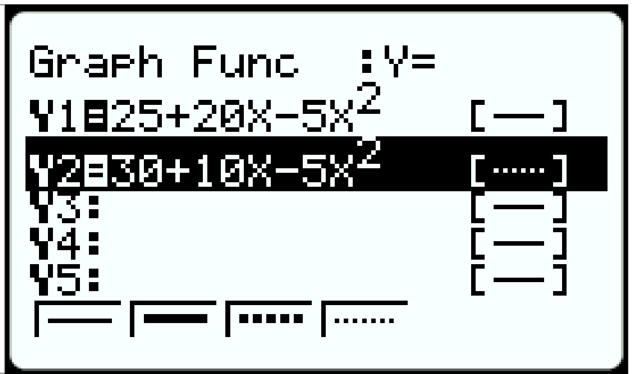
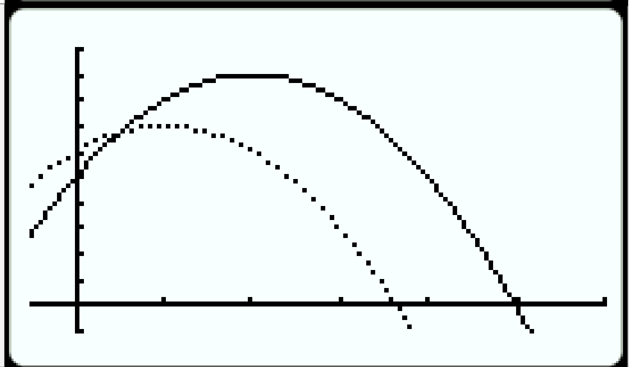
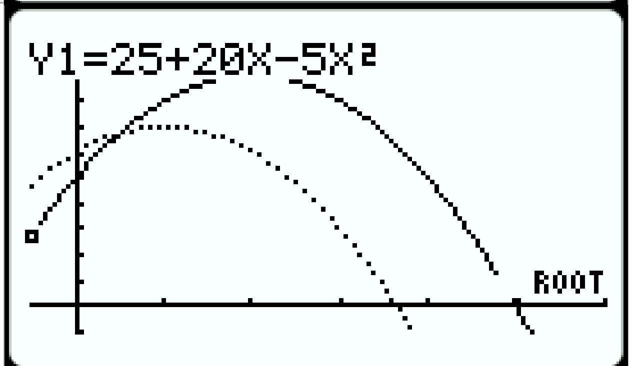
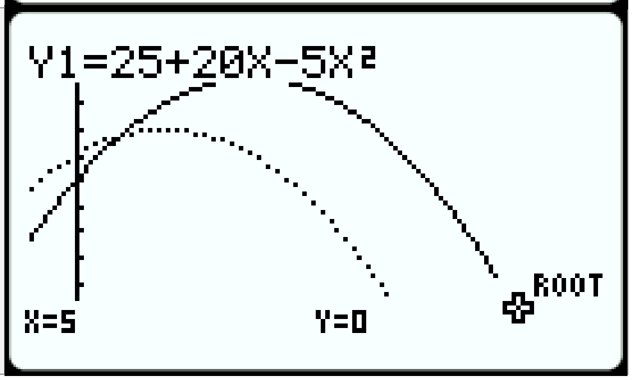


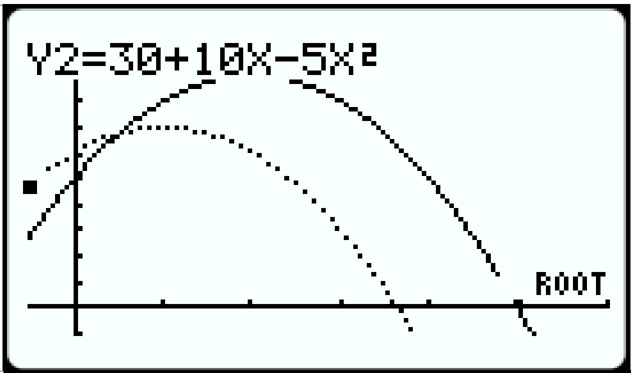
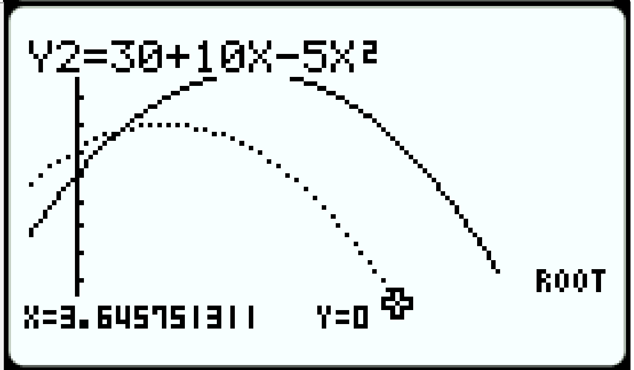
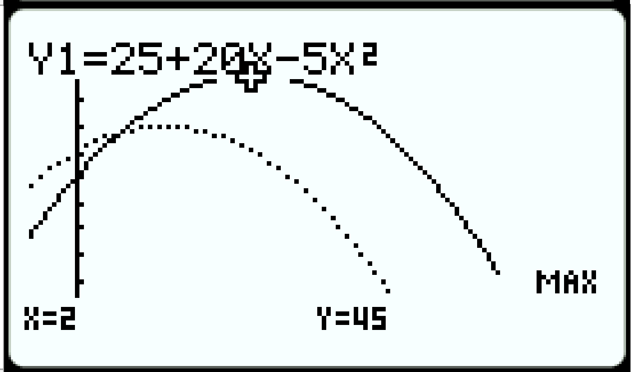
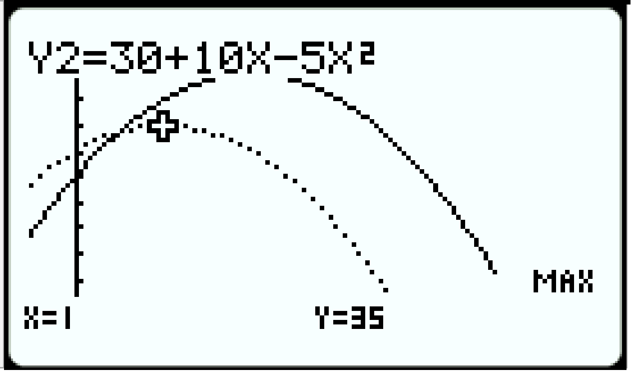
3. Being **projectiles**, we mainly want to focus on the equation's behavior in the **1st quadrant**. The settings to the right will give an optimal window for this problem. When finished, press [EXE]. In practice, it may take students a few times to view the graph and **adjust the window** as necessary. Minor **translation adjustments** to the graph can also be made utilizing the **arrow keys**.



4. Enter the motion equations for **Object A** and **Object B** for **Y1** and **Y2**. Use "*x*" as your input variable instead of "*t*". Press as you finish each equation. To more easily see which graph is which, press [F4] – [STYL] to change the **line style**.



5. Now press F4 – [·······] to change **Y2** to a **dotted line**.	
6. Press EXE to view the graphs of **Object A** (solid) and **Object B** (dotted). Even thoug**h** it starts at a higher height, **Object B reaches the ground first**. To find the specific times, press F5 – G-SLV to view the **Graph-Solve** options.	
7. To find the positive ***x*-intercepts** or **roots**, now press F1 – ROOT. Since there are 2 graphs, we need to select the graph. You will see the graph currently selected at the top of the screen with a point highlighted on that graph near the left.	
8. Press EXE to select **Y1**, Object A, and the **root**, **5** will display at the bottom of the screen. This shows that **Object A** hit the ground after **5 seconds**.	

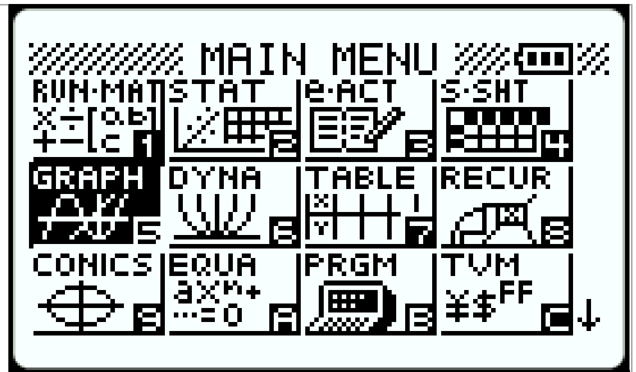
9. To find the time **Object B** hit the ground, press F5 – G-SLV and then F1 – ROOT again. This time select **Y2** by pressing the **up arrow** ⏶ so **Y2** is shown at the top of the screen**,** and the point is shown on the dotted graph of **Object B**.	
10. Press EXE to select **Y2**, Object B, and the **root**, **3.64575…** will display at the bottom of the screen. This shows that **Object A** hit the ground after **about 3.65 seconds**.	
11. To find the **maximum height** of each object, press F5 – G-SLV again. However, this time, press F2 – MAX. Like the **root function**, we need to select which graph. Press EXE to select **Y1**, Object A, and the **max point**, **(2,45)**, will display at the bottom of the screen. This shows that **Object A** reached a **maximum height** of **45 meters** after **2 seconds** in the air.	
12. Repeat the steps above to find the maximum height for **Object B**. Use the up arrow this time to select **Y2**. Press EXE and the **max point**, **(1,35)**, will display at the bottom of the screen. This shows that **Object B** reached a **maximum height** of **35 meters** after **1 second** in the air.	

Lesson 20 – Graphing to Predict if Zeros are Rational or Irrational

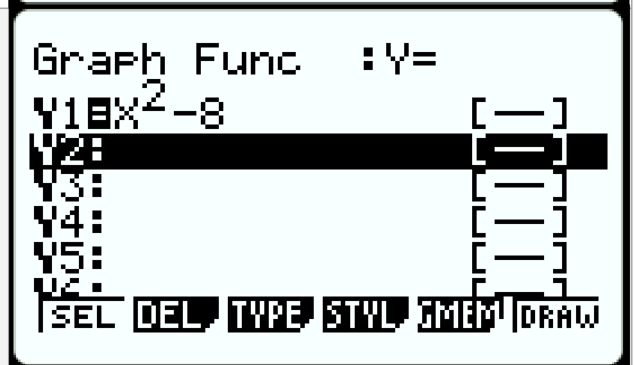
(Example: IM Lesson 20.2: Suspected Irrational Solutions)

1. This task requires students to **find zeros** of 4 different quadratic functions graphically and **predict** if the zeros are **rational** or **irrational**.

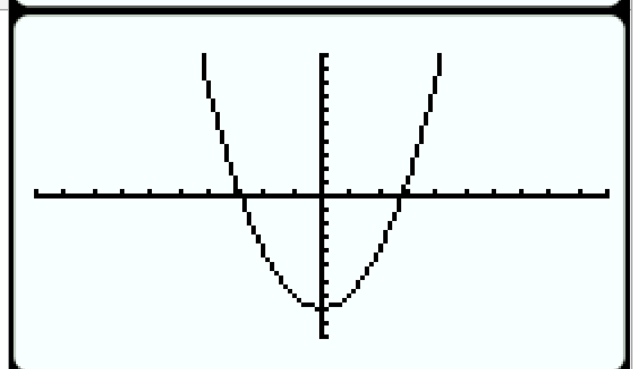
   To graph each equation, press [MENU], [5] - GRAPH.



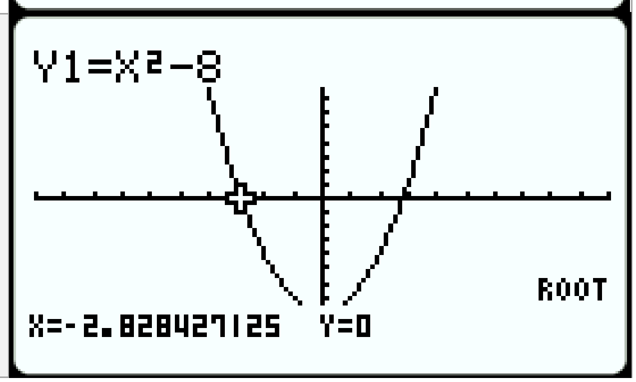
2. Type the first function, $x^2 - 8$, in for **Y1**.

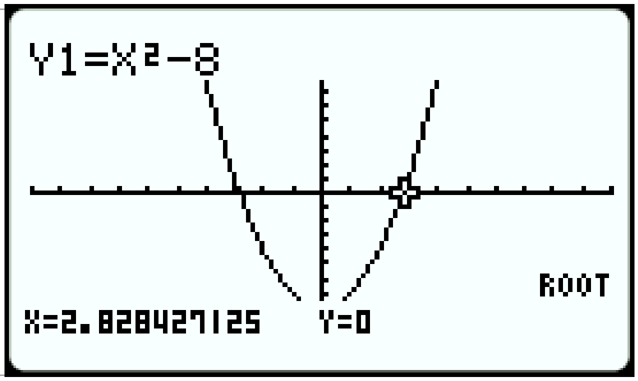


3. Press [F6]–DRAW to see the graph of the function.



4. We can see the graph crossing the ***x*-axis** twice, yielding 2 solutions. To find the **zeros**, or **roots**, press [F5]–G-SLV followed by [F1]–ROOT and the **first root** will appear at the bottom of the screen. The **first zero** is approximately **-2.828427125**. To find the **second root**, press the **right arrow** ▶.



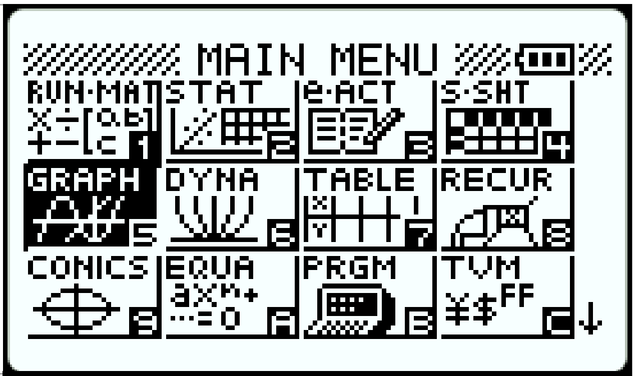
5. The **second zero** is around **2.828427125**. Since neither zero terminates no**r** repeats, students can **assume** these are both **irrational**. To know with 100% certainty, **solve algebraically** or utilize the **polynomial solver**. In this case, either method would show the exact answer of $\boldsymbol{x = \pm\sqrt{8}}$. To find the zeros for the other 3 functions, press EXIT, then repeat the previous steps.	

Lesson 22 – Verifying Quadratic Expressions Written in Vertex Form

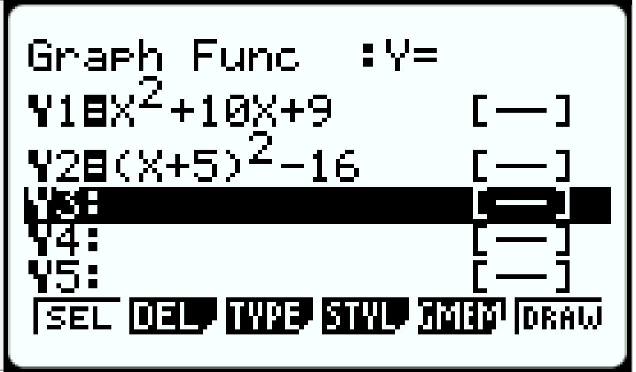
(Example: IM Lesson 22.2: Back and Forth.)

1. We can use the graphing function on our calculator to **verify** if the **vertex form** of a quadratic function correctly matches the original **standard form**.

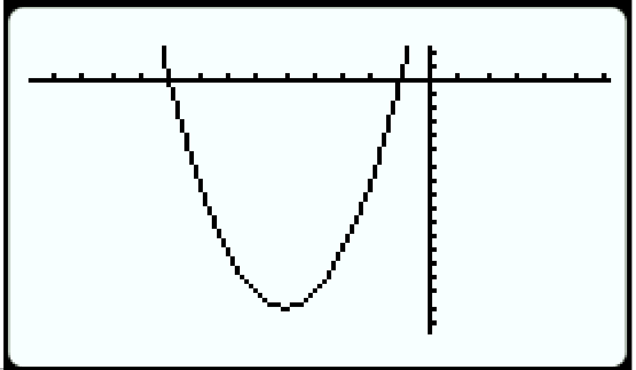
   Go to the **Graph App** to graph each equation; press [MENU], [5] - GRAPH.



2. **Equivalent expressions** will have graphs that will be identical. Equal inputs to equivalent expressions will have equal outputs. Graphically, this means the two graphs will **completely overlap** each other. Type the two quadratic forms in for **Y1** and **Y2**. When each is complete, press [EXE].



3. Press [F6] – DRAW to see if the graphs overlap. It looks as if there is **only one** function that is graphed, therefore the functions are **equivalent**!



4. We can further verify the graphs overlap by looking at their table. Press [MENU], [7] - TABLE to open the **Table App**. The same functions will appear there for **Y1** and **Y2**. Press [F6] – TABL to view their tables, observing equal outputs for each input.